Reduced-Rank Techniques for Sensor Array Signal Processing and Communications : Design, Algorithms and Applications

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Outline

Part I :

– Introduction

– System model and rank reduction

– Reduced-rank MMSE and MVDR designs

– Eigen-decomposition techniques

– Krylov subspace techniques

Outline (continued)

Part II :

- Joint and iterative optimization (JIO) techniques
- Joint interpolation, decimation and filtering (JIDF) techniques
- Model order selection
- Applications, perspectives and future work
- Concluding remarks

Introduction

- Reduced-rank detection and estimation techniques are fundamental tools in signal processing and communications.
- Motivation of reduced-rank signal processing :
  - robustness against noise and model uncertainties,
  - computational efficiency,
  - decompositions of signals for design and analysis,
  - inverse problems,
  - feature extraction,
  - dimensionality reduction,
  - problems with short data record, faster training .

Introduction

- Main goals of reduced-rank methods :
  - simplicity, ease of deployment,
  - to provide minimal reconstruction error losses,
  - to allow simple mapping and inverse mapping functions,
  - to improve convergence and tracking performance for dynamic signals,
  - to reduce the need for storage of coefficients,
  - to provide amenable and stable implementation,

Introduction

- Communications :
  - Interference mitigation, synchronization, fading mitigation, channel estimation.
  - Estimation with MMSE or LS criteria (Haykin [1]) :

    $$\mathbf{w} = \mathbf{R}^{-1}\mathbf{p},$$

    where

    $\mathbf{w}$ is a parameter vector with M coefficients,

    $\mathbf{r}(i)$ is the $M \times 1$ input data vector,

    $\mathbf{R} = E[\mathbf{r}(i)\mathbf{r}^H(i)]$ is the $M \times M$ covariance matrix,

    $\mathbf{p} = E[d^*(i)\mathbf{r}(i)]$ and $d(i)$ is the desired signal.

  - Detection approaches using MMSE or LS estimates.
  - Problems : dimensionality of system, matrix inversions.
  - How to improve performance ?
  - How do we deal with the computational complexity ?

Introduction

- Array signal processing :
  - Beamforming, direction finding, information combining with sensors, radar and sonar (van Trees [2]).
  - Parameter estimation with the MVDR criterion :

$$\mathbf{w} = \xi^{-1}\mathbf{R}^{-1}\mathbf{a}(\Theta_k),$$

    where

    $\mathbf{w}$ is a parameter vector with M coefficients,

    $\mathbf{r}(i)$ is the $M \times 1$ input data vector,

    $\mathbf{R} = E[\mathbf{r}(i)\mathbf{r}^H(i)]$ is the $M \times M$ covariance matrix,

    $\mathbf{a}(\Theta_k)$ is the $M \times 1$ array response vector and

    $\xi = \mathbf{a}(\Theta_k)^H\mathbf{R}^{-1}\mathbf{a}(\Theta_k)$.
  - Use of MVDR for beamforming and direction finding.
- Any idea ?
- Undermodelling ? $\rightarrow$ designer has to select the key features of $\mathbf{r}(i)$ $\rightarrow$ reduce-rank signal processing

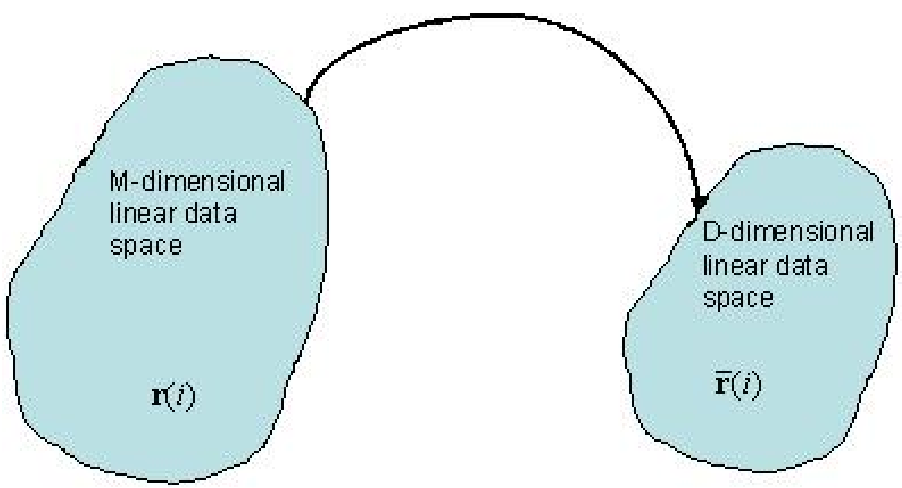
System Model and Rank Reduction

– Consider the following linear model

$$\mathbf{r}(i) = \mathbf{H}(i)\mathbf{s}(i) + \mathbf{n}(i),$$

where $\mathbf{s}(i)$ is a $M \times 1$ discrete-time signal organized in data vectors, $\mathbf{r}(i)$ is the $M \times 1$ input data, $\mathbf{H}(i)$ is a $M \times M$ matrix and $\mathbf{n}(i)$ is $M \times 1$ noise vector.

– Dimensionality reduction $\rightarrow$ an M-dimensional space is mapped into a D-dimensional subspace.



System Model and Rank Reduction

– A general reduced-rank version of $\mathbf{r}(i)$ can be obtained using a transformation matrix $\mathbf{S}_D$ (assumed fixed here) with dimensions $M \times D$, where D is the rank. Please see Haykin [1], Scharf-91 [3], Scharf and Tufts-87 [4], Scharf and van Veen-87[5].

– In other words, the mapping is carried out by the transformation matrix $\mathbf{S}_D$.

– The resulting reduced-rank observed data is given by

$$\bar{\mathbf{r}}(i) = \mathbf{S}_D^H \mathbf{r}(i)$$

where $\bar{\mathbf{r}}(i)$ is a $D \times 1$ vector.

– Challenge : How to efficiently (or optimally) design $\mathbf{S}_D$?

Historical Overview of Reduced-Rank Methods

– Origins of reduced-rank methods as a structured field :

- 1987 - Louis Scharf from University of Colorado defined the problem as “a transformation in which a data vector can be represented by a reduced number of effective features and yet retain most of the intrinsic information of the input data” Scharf and Tufts-87 [4], Scharf and van Veen-87[5].

- 1987- Scharf - Investigation and establishment of the bias versus noise variance trade-off.

Historical Overview of Reduced-Rank Methods

- Early Methods :
  - Hotelling and Eckhart (see Scharf [3]) in the 1930's $\rightarrow$ first methods using eigen-decompositions or principal components.
  - Early 1990's - applications of eigen-decomposition techniques for reduced-rank estimation in communications. See Haimovich and Bar-Ness [7], Wang and Poor [8], and Hua *e*t al. [9].
  - 1994 $\rightarrow$ Cai and Wang [6], Bell Labs : joint domain localised adaptive processing $\rightarrow$ radar-based scheme, medium complexity.
- Main problems of eigen-decomposition techniques :
  - Require computationally expensive SVD or EVD, or algorithms to obtain the eigenvalues and eigenvectors.
  - Performance degradation with the increase in the signal subspace.

Historical Overview of Reduced-Rank Methods

– 1997 - Goldstein and Reed [10], University of Southern California : cross-spectral approach.

  - Appropriate selection of eigenvalues values which addresses the performance degradation.
  - Remaining problem : EVD or SVD requirement.

– 1998/9 - Partial despreading (PD) of Singh and Milstein [19], University of California at San Diego :

  - simple but suboptimal and restricted to CDMA multiuser detection.

Historical Overview of Reduced-Rank Methods

- Krylov subspace methods : conjugate gradient techniques developed in the 1950s.
- 1997 $\rightarrow$ Pados and Batallama [20]-[24], University of New York, Buffalo : auxiliary vector filtering (AVF) algorithm :
  - do not require SVD .
  - very fast convergence but complexity is still a problem.
- 1997 - 2004 - multistage Wiener filter (MSWF) of Goldstein, Reed and Scharf and its variants [12]-[16] :
  - State-of-the-art in the field and benchmark.
  - Very fast convergence, rank not scaling with system size.
  - equivalence between the AVF (with orthogonal AVs) and the MSWF was established by Chen, Mitra and Schniter [17].
  - Complexity is still a problem as well as the existence of numerical instability for implementation.

Historical Overview of Reduced-Rank Methods

- 2004 $\rightarrow$ de Lamare and Sampaio-Neto ([27])- Interpolated FIR filters with time-varying interpolators : low complexity, good performance but rank limited.
- 2005 $\rightarrow$ de Lamare and Sampaio-Neto - Joint interpolation, decimation and filtering (JIDF) scheme [33]-[35] - Best known scheme, flexible, smallest complexity in the field, patented.
- 2007 $\rightarrow$ de Lamare, Haardt and Sampaio-Neto - Robust MSWF [17] - Development of a robust version of the MSWF using the constrained constant modulus (CCM) design criterion.
- 2007 $\rightarrow$ de Lamare and Sampaio-Neto - Joint iterative optimisation of filters - (JIO) - Development of a generic reduced-rank scheme that is very good for mapping and inverse mapping [28].
- 2008 $\rightarrow$ de Lamare, Sampaio-Neto and Haardt [37] - Robust JIDF-type approach called BARC - Development of a robust version of the JIDF using the CCM design criterion.

Linear MMSE Reduced-Rank Estimator Design

– The linear MMSE estimator is the vector $\mathbf{w} = \left[w_1\ w_2\ \ldots\ w_M\right]^T$, which is designed to minimize the MSE cost function

$$J = E\left[|d(i) - \mathbf{w}^H \mathbf{r}(i)|^2\right]$$

where $d(i)$ is the desired signal.

– The solution is $\mathbf{w} = \mathbf{R}^{-1}\mathbf{p}$, where $E[d^*(i)\mathbf{r}(i)]$ and $\mathbf{R} = E[\mathbf{r}(i)\mathbf{r}^H(i)$

– The estimator $\mathbf{w}$ can be also be computed via adaptive algorithms, however ...

– The convergence speed and tracking of these algorithms depends on M and the eigenvalue spread. Thus, large M implies slow convergence.

– Reduced-rank schemes circumvent these limitations via a reduction in the number of coefficients and the extraction of the key features of the data.

Linear MMSE Reduced-Rank Estimator Design

– Consider a reduced-rank input vector $\bar{\mathbf{r}}(i) = \mathbf{S}_D^H \mathbf{r}(i)$ as the input to an estimator represented by the D vector $\bar{\mathbf{w}} = \left[\bar{w}_1 \ \bar{w}_2 \ \ldots \bar{w}_D\right]^T$ for time interval i.

– The estimator output is

$$x(i) = \bar{\mathbf{w}}^H \mathbf{S}_D^H \mathbf{r}(i)$$

– The MMSE design problem can be stated as

$$\begin{aligned} \text{minimize} \quad \mathcal{J}(\bar{\mathbf{w}}) &= E\Big[|d(i) - x(i)|^2\Big] \\ &= E\Big[|d(i) - \bar{\mathbf{w}}^H \mathbf{S}_D^H \mathbf{r}(i)|^2\Big] \end{aligned}$$

where $d(i)$ is the desired signal.

Linear MMSE Reduced-Rank Estimator Design

– The MMSE design with the reduced-rank parameters yields

$$\bar{\mathbf{w}} = \bar{\mathbf{R}}^{-1}\bar{\mathbf{p}},$$

where
$\bar{\mathbf{R}} = E[\bar{\mathbf{r}}(i)\bar{\mathbf{r}}^H(i)] = \mathbf{S}_D^H\mathbf{R}\mathbf{S}_D$ is the reduced-rank covariance matrix,
$\mathbf{R} = E[\mathbf{r}(i)\mathbf{r}^H(i)]$ is the full-rank covariance matrix,
$\bar{\mathbf{p}} = E[d^*(i)\bar{\mathbf{r}}(i)] = \mathbf{S}_D^H\mathbf{p}$ and $\mathbf{p} = E[d^*(i)\mathbf{r}(i)]$.

– The associated MMSE for a rank D estimator is expressed by

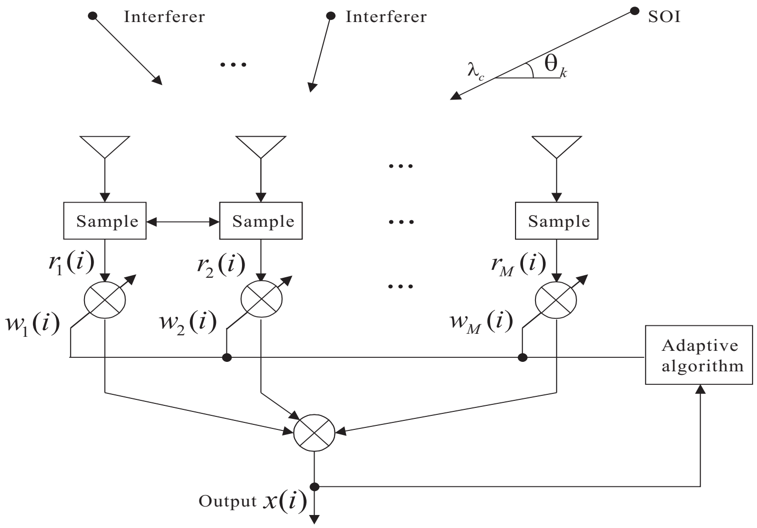
$$\text{MMSE} = \sigma_d^2 - \bar{\mathbf{p}}^H\bar{\mathbf{R}}^{-1}\bar{\mathbf{p}} = \sigma_d^2 - \mathbf{p}^H\mathbf{S}_D(\mathbf{S}_D^H\mathbf{R}\mathbf{S}_D)^{-1}\mathbf{S}_D^H\mathbf{p}$$

where σ_d^2 is the variance of $d(i)$.

MVDR Reduced-Rank Beamformer Design

– Consider a uniform linear array (ULA) of M elements.

– There are K narrowband sources impinging on the array ($K < M$) with directions of arrival (DOA) θ_k for $k = 1, 2, \ldots, K$.



– The received signal is given by :

$$\boldsymbol{r}(i) = \sum_{k=1}^{K} \boldsymbol{a}(\theta_k) s_k(i) + \boldsymbol{n}(i)$$

– Reduced-rank array processing : The output of the array is

$$x(i) = \bar{\boldsymbol{w}}^H \bar{\boldsymbol{r}}(i) = \bar{\boldsymbol{w}}^H(i) \boldsymbol{S}_D^H \boldsymbol{r}(i)$$

MVDR Reduced-Rank Beamformer Design

– In order to design the reduced-rank beamformer $\bar{\boldsymbol{w}}(i)$ we consider the following optimization problem

$$\text{minimize } E\Big[|\bar{\boldsymbol{w}}^H \boldsymbol{S}_D^H \boldsymbol{r}(i)|^2\Big] = \bar{\boldsymbol{w}}^H \boldsymbol{S}_D^H \boldsymbol{R} \boldsymbol{S}_D \bar{\boldsymbol{w}}$$
$$\text{subject to } \bar{\boldsymbol{w}}^H \boldsymbol{S}_D^H \boldsymbol{a}(\theta_k) = 1$$

– Approach to obtain a solution : method of Lagrange multipliers

$$\mathcal{L}(\bar{\boldsymbol{w}}, \lambda) = E\Big[|\bar{\boldsymbol{w}}^H \boldsymbol{S}_D^H \boldsymbol{r}(i)|^2\Big] + \lambda(\bar{\boldsymbol{w}}^H \mathbf{S}_D^H \boldsymbol{a}(\theta_k) - 1)$$

– The solution to this design problem is

$$\bar{\boldsymbol{w}} = \frac{(\boldsymbol{S}_D^H \boldsymbol{R} \boldsymbol{S}_D)^{-1} \boldsymbol{S}_D^H \boldsymbol{a}(\theta_k)}{\boldsymbol{a}^H(\theta_k) \boldsymbol{S}_D(i) (\boldsymbol{S}_D^H \boldsymbol{R} \boldsymbol{S}_D)^{-1} \boldsymbol{S}_D^H \boldsymbol{a}(\theta_k)} = \frac{\bar{\boldsymbol{R}}^{-1} \bar{\boldsymbol{a}}(\theta_k)}{\bar{\boldsymbol{a}}^H(\theta_k) \bar{\boldsymbol{R}}^{-1} \bar{\boldsymbol{a}}(\theta_k)}$$

where the reduced-rank covariance matrix is $\bar{\boldsymbol{R}} = E[\bar{\boldsymbol{r}}(i)\bar{\boldsymbol{r}}^H(i)] = \boldsymbol{S}_D^H \boldsymbol{R} \boldsymbol{S}_D$ and the reduced-rank steering vector is $\bar{\boldsymbol{a}}(\theta_k) = \boldsymbol{S}_D^H \boldsymbol{a}(\theta_k)$.

MVDR Reduced-Rank Beamformer Design

– The associated minimum variance (MV) for an MVDR beamformer with rank D is

$$\begin{aligned}\mathrm{MV} &= \frac{1}{\bar{\boldsymbol{a}}^H(\theta_k)\bar{\boldsymbol{R}}^{-1}\bar{\boldsymbol{a}}(\theta_k)} \\ &= \frac{1}{\boldsymbol{a}(\theta_k)^H\boldsymbol{S}_D(\boldsymbol{S}_D^H\boldsymbol{R}\boldsymbol{S}_D)^{-1}\boldsymbol{S}_D^H\boldsymbol{a}(\theta_k)}\end{aligned}$$

– The above expression can be used for direction finding by replacing the angles θ_k with a time-varying parameter (ω) in order to scan the possible angles.

– It can also be employed for general applications of spectral estimation including spectral sensing.

Eigen-Decomposition Techniques

– Why are eigen-decomposition techniques used ?

– For MMSE parameter estimation and a rank D estimator we have

$$\text{MMSE} = \sigma_d^2 - \mathbf{p}^H \mathbf{S}_D (\mathbf{S}_D^H \mathbf{R} \mathbf{S}_D)^{-1} \mathbf{S}_D^H \mathbf{p}$$

– Taking the gradient of MMSE with respect to $\boldsymbol{S}_D$, we get

$$\boldsymbol{S}_{D,\text{opt}} = [\boldsymbol{v}_1 \ldots \boldsymbol{v}_D],$$

where $\boldsymbol{v}_d$ for $d = 1, \ldots, D$ are the eigenvectors of $\mathbf{R}$.

– For MV parameter estimation and a rank D estimator we have

$$\text{MV} = \frac{1}{\boldsymbol{a}(\theta_k)^H \boldsymbol{S}_D (\boldsymbol{S}_D^H \boldsymbol{R} \boldsymbol{S}_D)^{-1} \boldsymbol{S}_D^H \boldsymbol{a}(\theta_k)}$$

– Taking the gradient of MV with respect to $\boldsymbol{S}_D$, we get

$$\boldsymbol{S}_{D,\text{opt}} = [\boldsymbol{v}_1 \ldots \boldsymbol{v}_D]$$

Eigen-Decomposition Techniques

– Rank reduction is accomplished by eigen- decomposition on the input data covariance matrix

$$\boldsymbol{R} = \boldsymbol{V}\boldsymbol{\Lambda}\boldsymbol{V}^H,$$

where
$\boldsymbol{V} = [\boldsymbol{v}_1 \ldots \boldsymbol{v}_M]$ and
$\boldsymbol{\Lambda} = \text{diag}(\lambda_1, \ldots, \lambda_M)$.

– Early techniques : selection of eigenvectors $\boldsymbol{v}_j$ $(j = 1, \ \ldots, M)$ corresponding to the largest eigenvalues λ_j
$\rightarrow$ Transformation matrix is

$$\boldsymbol{S}_D(i) = [\boldsymbol{v}_1 \ldots \boldsymbol{v}_D]$$

Eigen-Decomposition Techniques

– Cross-spectral approach of Goldstein and Reed : choose eigen-vectors that minimise the design criterion
$\rightarrow$ Transformation matrix is

$$\boldsymbol{S}_D(i) = [\boldsymbol{v}_i \ldots \boldsymbol{v}_t]$$

– Problems : Complexity $O(M^3)$, optimality implies knowledge of $\boldsymbol{R}$ but this has to be estimated.

– Complexity reduction : adaptive subspace tracking algorithms (popular in the end of the 90s) but still complex and susceptible to tracking problems.

– Can we skip or circumvent an eigen-decomposition ?

Krylov Subspace Techniques

- Krylov subspace techniques have a rich history in solving systems of equations and in numerical linear algebra.
- In array signal processing and communications, we are usually interested in solving symmetric and positive definite systems.
- In this context, one of the most important Krylov subspace methods is the conjugate gradient technique invented by Hestenes and Stiefel in 1950s.
- Main idea : to solve $\boldsymbol{R}\boldsymbol{w} = \boldsymbol{p}$ in the Krylov subspace that spans after k iterations span$\{\boldsymbol{p}, \boldsymbol{R}\boldsymbol{p}, \ldots, \boldsymbol{R}^{k-1}\boldsymbol{p}\}$.
- Complexity : quadratic in M.

Conjugate Gradient Techniques

– Main idea : to solve $\boldsymbol{R}\boldsymbol{w} = \boldsymbol{p}$ in k iterations.
  Initialize all parameter vectors $\boldsymbol{g}_o = \boldsymbol{p} - \boldsymbol{R}\boldsymbol{w}_0$, $\boldsymbol{w}_0$, $\boldsymbol{v}_0$
  For $k = 1, \ldots, K$ do :
  – Calculate step size :

$$\alpha_k = \frac{\boldsymbol{g}_{k-1}^H \boldsymbol{g}_{k-1}}{\boldsymbol{p}_k^H \boldsymbol{R} \boldsymbol{p}_k}$$

  – Compute parameters :

$$\boldsymbol{w}_k = \boldsymbol{w}_{k-1} + \alpha \boldsymbol{v}_k$$

  – Calculate step size :

$$\beta_k = \frac{\boldsymbol{v}_{k-1}^H \boldsymbol{R} \boldsymbol{g}_{k-1}}{\boldsymbol{v}_{k-1}^H \boldsymbol{R} \boldsymbol{v}_{k-1}}$$

  – Compute direction vectors :

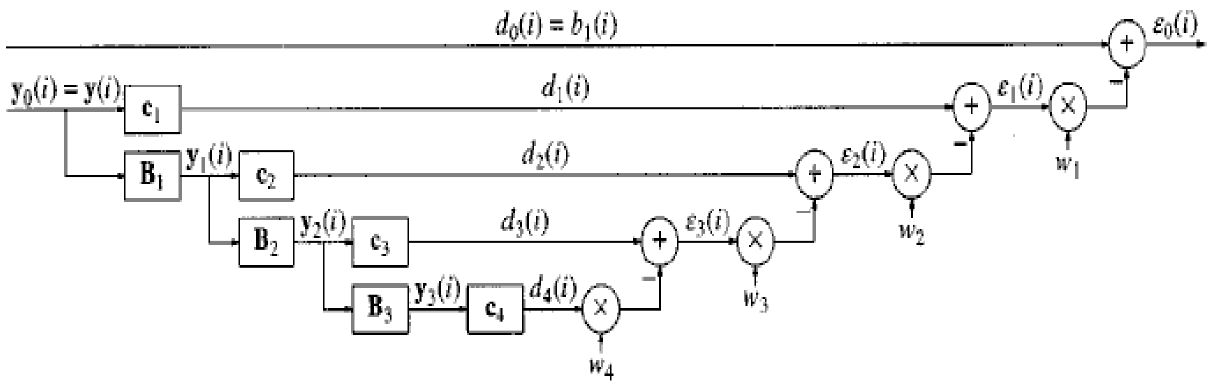
$$\boldsymbol{v}_k = \boldsymbol{g}_k + \beta_k \boldsymbol{v}_{k-1}$$

  – Calculate negative gradient :

$$\boldsymbol{g}_k = \boldsymbol{g}_{k-1} - \alpha_k \boldsymbol{R} \boldsymbol{v}_k$$

Multi-stage Wiener Filter

– Rank reduction is accomplished by a successive refinement procedure that generates a set of basis vectors, i.e. the signal subspace, known in numerical analysis as the Krylov subspace.



– Design : use of nested filters $\boldsymbol{c}_j$ $(j = 1, \ldots, M)$ and blocking matrices $\boldsymbol{B}_j$ for the decomposition $\rightarrow$ Projection matrix is

$$\boldsymbol{S}_D(i) = [\boldsymbol{p}, \boldsymbol{R}p, ..., \boldsymbol{R}^{D-1}\boldsymbol{p}]$$

– Advantages : rank D does not scale with system size, very fast convergence.

– Problems : complexity slightly inferior to RLS algorithms, not robust to signature mismatches in blind operation.

Robust Multi-stage Wiener Filter

– Rank reduction is accomplished by a similar successive refinement procedure to original MSWF. However, the design is based on the CCM criterion (de Lamare, Haardt and Sampaio-Neto, IEEE TSP, 2008)).

– Transformation matrix :

$$\mathbf{S}_D(i) = \Big[\mathbf{q}(i),\ \mathbf{R}(i)\mathbf{q}(i),\ \ldots,\ \mathbf{R}^{(D-1)}(i)\mathbf{q}(i)\Big]$$

– The reduced-rank CCM parameter vector with rank D is

$$\bar{\mathbf{w}}(i+1) = \Big(\mathbf{S}_D^H(i)\mathbf{R}(i)\mathbf{S}_D(i)\Big)^{-1}\mathbf{S}_D^H(i)\mathbf{q}(i),$$

where

$$\mathbf{q}(i) = \mathbf{d}(i) - (\mathbf{p}^H(i)\mathbf{R}^{-1}(i)\mathbf{p}(i))^{-1}(\mathbf{p}^H(i)\mathbf{R}^{-1}(i)\mathbf{d}(i) - \nu\ )\mathbf{p}(i),$$

$$\mathbf{d}(i) = E\Big[x^*(i)\boldsymbol{S}_D^H(i)\mathbf{r}(i)\Big]$$

Applications : Interference Suppression for CDMA

- We assess BER performance of the supervised LS, the CMV-LS and the CCM-LS and their full-rank and reduced-rank versions.

- The DS-CDMA system uses random sequences with $N = 64$.

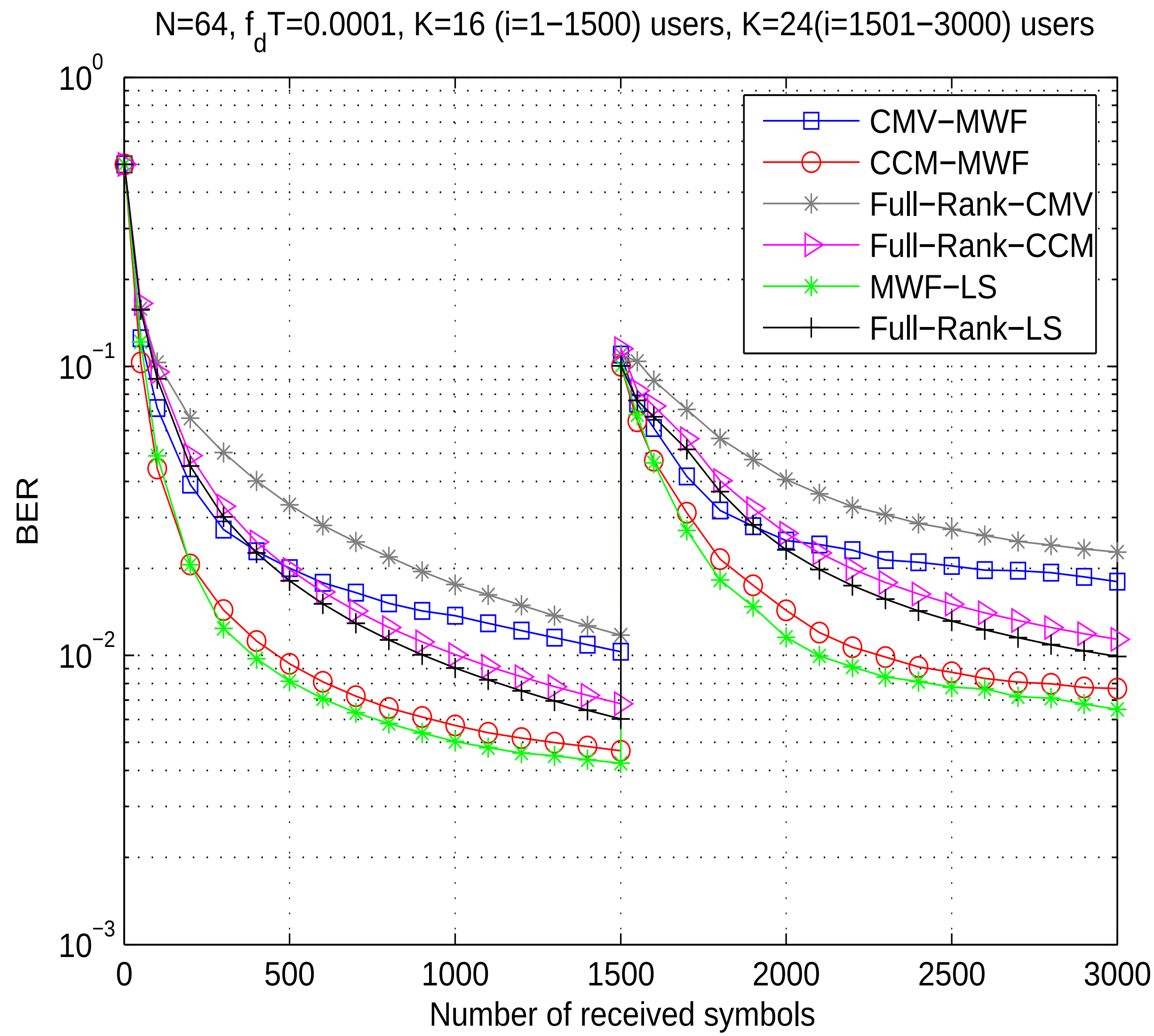
- We use 3-path channels with powers $p_{k,l}$ given by 0, -3 and -6 dB. In each run the spacing between paths is obtained from a discrete uniform random variable between 1 and 2 chips.

- Power distribution amongst the users : Follows a log-normal distribution with associated standard deviation of 1.5 dB.

- All LS type estimators use $\lambda = 0.998$ to ensure good performance and all experiments are averaged over 200 runs.

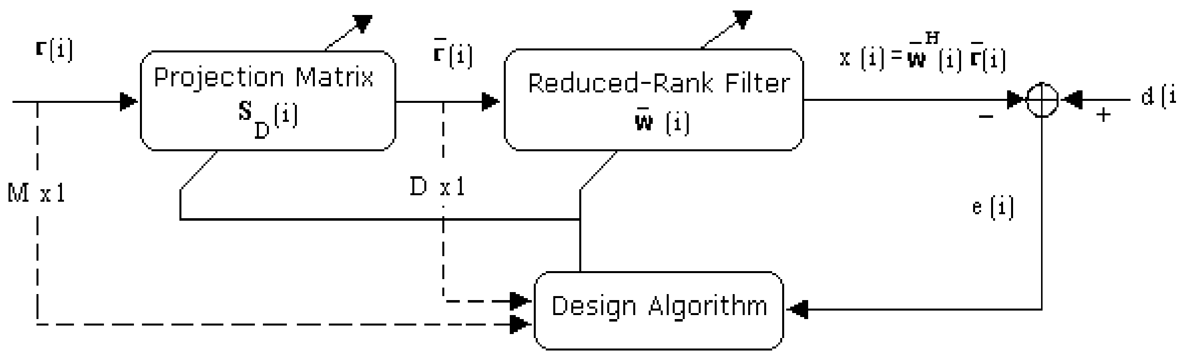
Applications : Interference Suppression for CDMA

– BER convergence performance at $E_b/N_0 = 12$ dB.



JIO Techniques

– Rank reduction is performed by joint and iterative optimisation (JIO) of the rank-reduction matrix $\boldsymbol{S}_D(i)$ and reduced-rank estimator $\bar{\boldsymbol{w}}(i)$.



– Design criteria : MMSE, LS, LCMV, etc

– Adaptive algorithms : LMS, RLS, etc

– Highlights : rank D does not scale with system size, very fast convergence, proof of global convergence established, very simple.

MMSE Design of JIO Scheme

– The MMSE expressions for the filters $\mathbf{S}_D(i)$ and $\bar{\mathbf{w}}(i)$ can be computed through the following cost function :

$$J = E\Big[|d(i) - \bar{\mathbf{w}}^H(i)\mathbf{S}_D^H(i)\mathbf{r}(i)|^2\Big]$$

– By fixing $\mathbf{S}_D(i)$ and minimizing the cost function with respect to $\bar{\mathbf{w}}(i)$, the reduced-rank estimator becomes

$$\bar{\mathbf{w}}(i) = \bar{\mathbf{R}}^{-1}(i)\bar{\mathbf{p}}(i)$$

where
$\bar{\mathbf{R}}(i) = E[\mathbf{S}_D^H(i)\mathbf{r}(i)\mathbf{r}^H(i)\mathbf{S}_D(i)] = E[\bar{\mathbf{r}}(i)\bar{\mathbf{r}}^H(i)]$,
$\bar{\mathbf{p}}(i) = E[d^*(i)\mathbf{S}_D^H(i)\mathbf{r}(i)] = E[d^*(i)\bar{\mathbf{r}}(i)]$.

MMSE Design of JIO Scheme

– Fixing $\bar{\mathbf{w}}(i)$ and minimizing the cost function with respect to $\mathbf{S}_D(i)$, we get

$$\mathbf{S}_D(i) = \mathbf{R}^{-1}(i)\mathbf{P}_D(i)\mathbf{R}_w^{-1}(i)$$

where
$\mathbf{R}(i) = E[\mathbf{r}(i)\mathbf{r}^H(i)]$,
$\mathbf{P}_D(i) = E[d^*(i)\mathbf{r}(i)\bar{\mathbf{w}}^H(i)]$ and
$\mathbf{R}_w(i) = E[\bar{\mathbf{w}}(i)\bar{\mathbf{w}}^H(i)]$.

– The associated MMSE is

$$\text{MMSE} = \sigma_d^2 - \bar{\mathbf{p}}^H(i)\bar{\mathbf{R}}^{-1}(i)\bar{\mathbf{p}}(i)$$

where $\sigma_d^2 = E[|d(i)|^2]$.

MMSE Design of JIO Scheme

- The filter expressions for $\bar{\mathbf{w}}(i)$ and $\mathbf{S}_D(i)$ are functions of one another and thus it is necessary to iterate them with an initial guess to obtain a solution.

- Unlike prior art, the JIO scheme provides an iterative exchange of information between the reduced-rank estimator and the transformation matrix.

- The key strategy lies in the joint optimization of the filters $\rightarrow$ the method is guided by the optimization algorithm.

- The rank D or model order must be set by the designer to ensure appropriate or adjusted on-line.

Adaptive JIO-LMS Algorithm

Initialize all parameter vectors, dimensions

For each data vector $i = 1, \ldots, Q$ do :

– Perform dimensionality reduction :

$$\bar{\boldsymbol{r}}(i) = \boldsymbol{S}_D^H(i)\boldsymbol{r}(i)$$

– Estimate parameters

$$\boldsymbol{S}_D(i+1) = \boldsymbol{S}_D(i) + \eta(i)e^*(i)\boldsymbol{r}(i)\bar{\boldsymbol{w}}^H(i)$$

$$\bar{\boldsymbol{w}}(i+1) = \bar{\boldsymbol{w}}(i) + \mu(i)e^*(i)\bar{\boldsymbol{r}}(i)$$

where $e(i) = d(i) - \bar{\boldsymbol{w}}^H(i)\boldsymbol{S}_D^H(i)\boldsymbol{r}(i)$.

Applications : Interference Suppression for CDMA

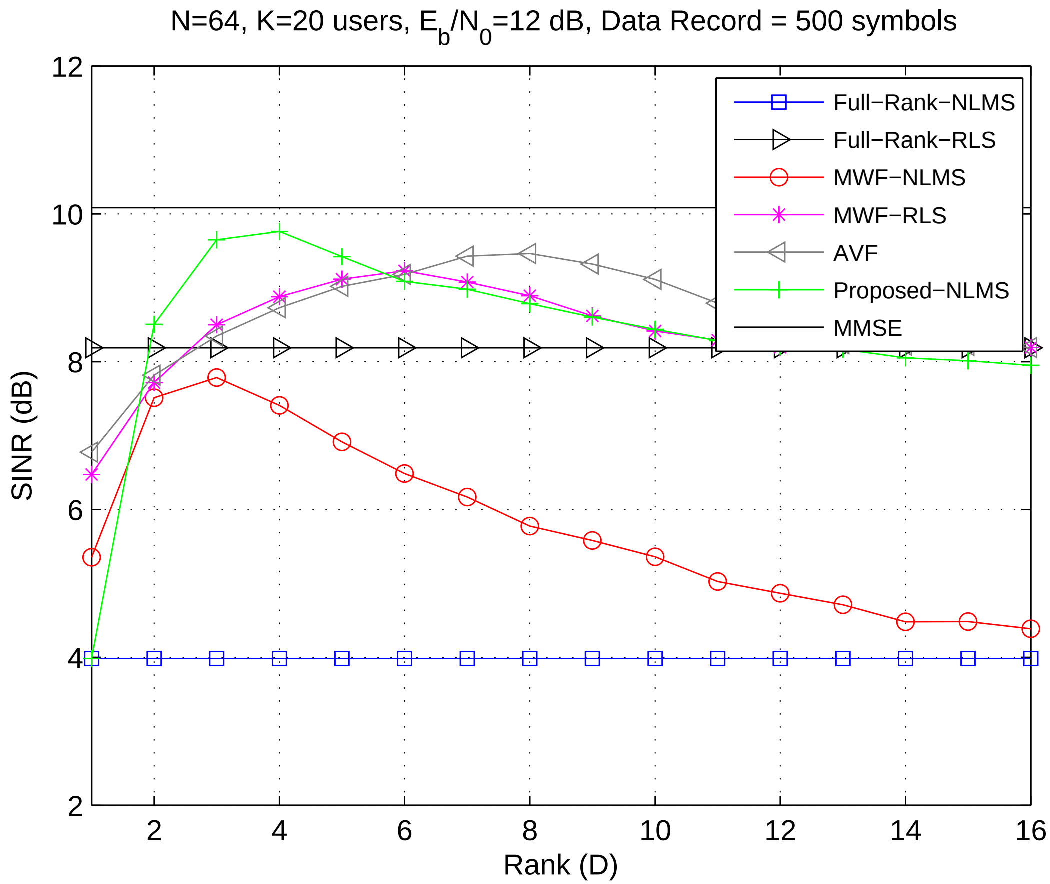
– We consider the uplink of a symbol synchronous BPSK DS-CDMA system with K users, N chips per symbol and L propagation paths.

– Initialization : for all simulations, we use $\bar{\mathbf{w}}(0) = \mathbf{0}_{D,1}$, $\mathbf{S}_D(0) = [\mathbf{I}_D \ \mathbf{0}_{D,M-D}]^T$.

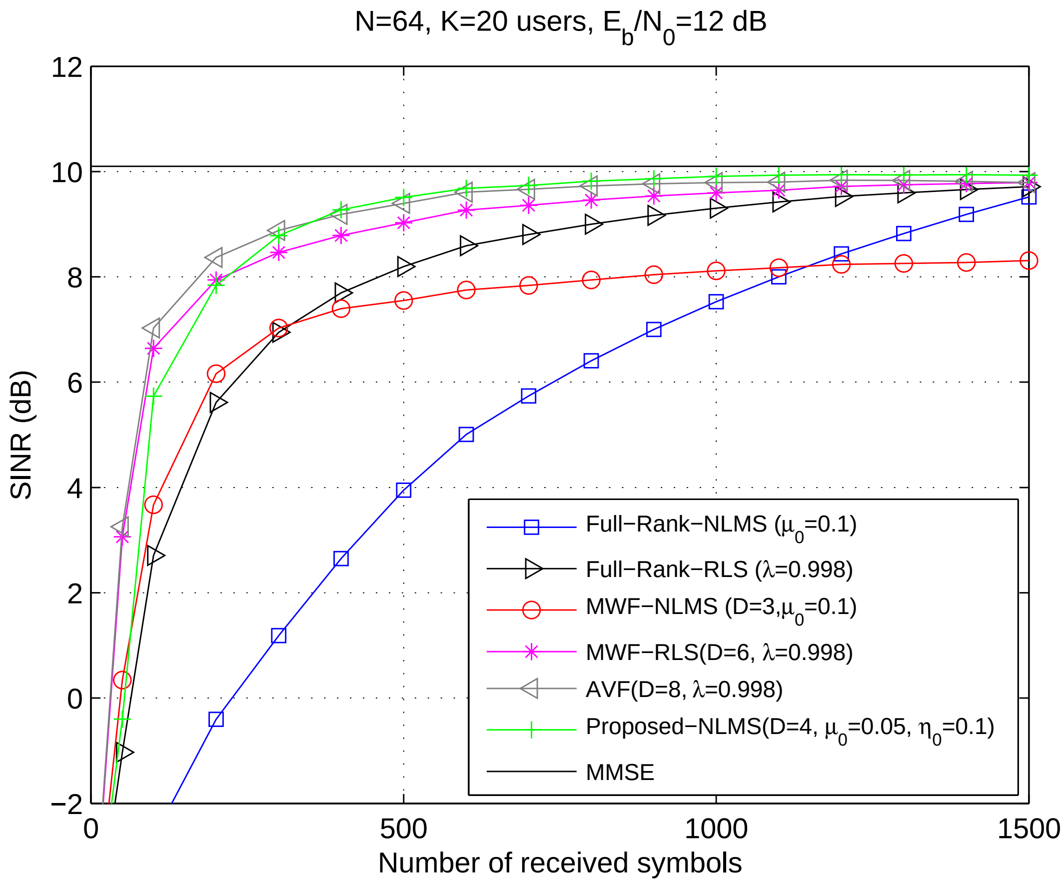
– We assume $L = 9$ as an upper bound on the channel delay spread, use 3-path channels with relative powers given by 0, -3 and -6 dB, where in each run the spacing between paths is obtained from a discrete uniform random variable between 1 and 2 chips and average the experiments over 200 runs.

– The system has a power distribution amongst the users for each run that follows a log-normal distribution with associated standard deviation equal to 1.5 dB.

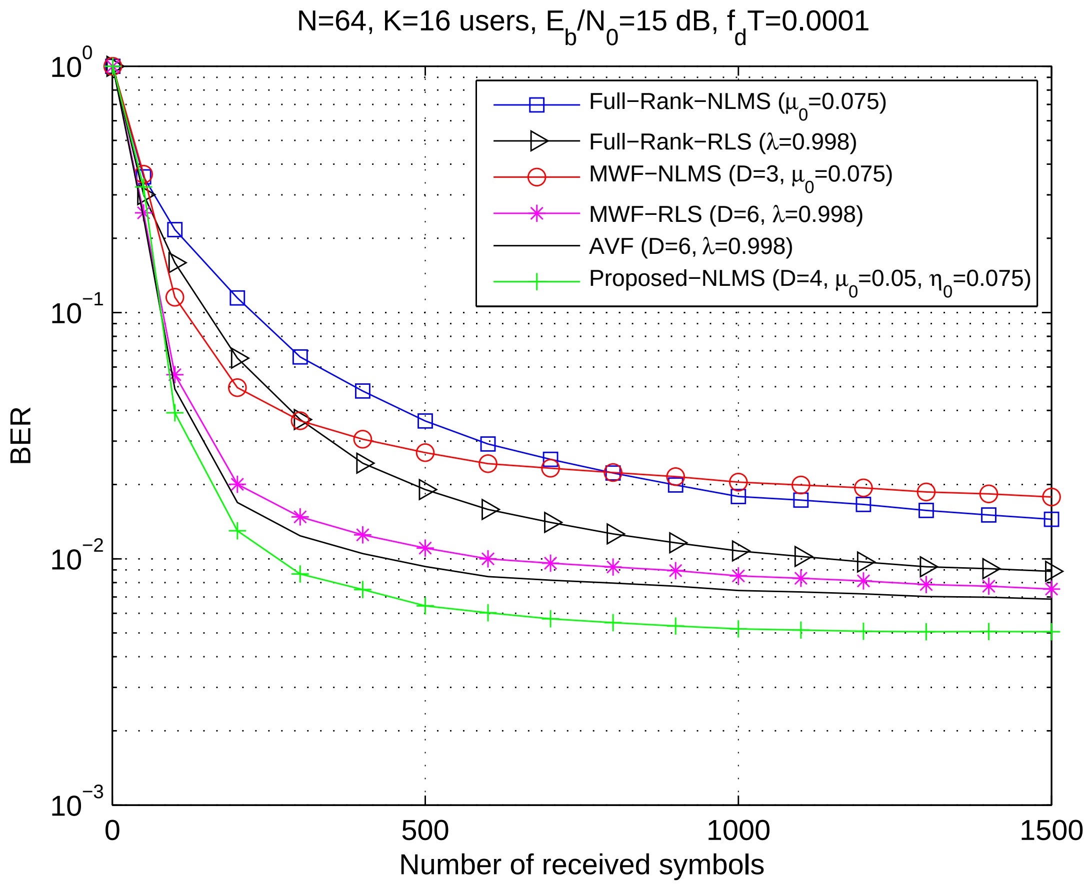
Applications : Interference Suppression for CDMA



Applications : Interference Suppression for CDMA



Applications : Interference Suppression for CDMA



MVDR Design of JIO Scheme

– Main differences in approach : the beamformers $\mathbf{S}_D(i)$ and $\bar{\mathbf{w}}(i)$ are jointly optimized and certain key quantities are assumed statistically independent.

– The MVDR expressions for the beamformers $\mathbf{S}_D(i)$ and $\bar{\mathbf{w}}(i)$ can be computed via the proposed optimization problem

$$\text{minimize } E\Big[|\bar{\boldsymbol{w}}^H(i)\boldsymbol{S}_D^H(i)\boldsymbol{r}(i)|^2\Big] = \bar{\boldsymbol{w}}^H(i)\boldsymbol{S}_D^H(i)\boldsymbol{R}\boldsymbol{S}_D(i)\bar{\boldsymbol{w}}(i)$$
$$\text{subject to } \bar{\boldsymbol{w}}^H(i)\boldsymbol{S}_D^H(i)\boldsymbol{a}(\theta_k) = 1$$

– Solution $\rightarrow$ method of Lagrange multipliers

$$\mathcal{L}(\boldsymbol{S}_D(i), \bar{\boldsymbol{w}}(i), \lambda) = E\Big[|\bar{\boldsymbol{w}}^H(i)\boldsymbol{S}_D^H(i)\boldsymbol{r}(i)|^2\Big] + \lambda(\bar{\boldsymbol{w}}^H(i)\mathbf{S}_D^H(i)\boldsymbol{a}(\theta_k) - 1)$$

MVDR Design of JIO Scheme

– By fixing $\bar{\boldsymbol{w}}(i)$, minimizing $\mathcal{L}(\boldsymbol{S}_D(i), \bar{\boldsymbol{w}}(i), \lambda)$ with respect to $\boldsymbol{S}_D(i)$ and solving for λ, we get

$$\boldsymbol{S}_D(i) = \frac{\boldsymbol{R}^{-1}\boldsymbol{a}(\theta_k)\bar{\boldsymbol{w}}^H(i)\boldsymbol{R}_w^{-1}}{\bar{\boldsymbol{w}}^H(i)\boldsymbol{R}_w^{-1}\bar{\boldsymbol{w}}(i)\boldsymbol{a}^H(\theta_k)\boldsymbol{R}^{-1}\boldsymbol{a}(\theta_k)},$$

where
$\boldsymbol{R} = E[\boldsymbol{r}(i)\boldsymbol{r}^H(i)]$ and
$\boldsymbol{R}_w = E[\bar{\boldsymbol{w}}(i)\bar{\boldsymbol{w}}^H(i)]$.

– A simplified expression for $\boldsymbol{S}_D(i)$ obtained analytically with the exploitation of the constraint is given by

$$\boldsymbol{S}_D(i) = \frac{\boldsymbol{P}(i)\boldsymbol{a}(\theta_k)\bar{\boldsymbol{a}}^H(\theta_k)}{\boldsymbol{a}^H(\theta_k)\boldsymbol{P}(i)\boldsymbol{a}(\theta_k)}$$

MVDR Design of JIO Scheme

– By fixing $\boldsymbol{S}_D(i)$, minimizing the Lagrangian with respect to $\bar{\boldsymbol{w}}(i)$ and solving for λ, we arrive at the expression for $\bar{\boldsymbol{w}}(i)$

$$\bar{\boldsymbol{w}}(i) = \frac{\bar{\boldsymbol{R}}^{-1}(i)\bar{\boldsymbol{a}}(\theta_k)}{\bar{\boldsymbol{a}}^H(\theta_k)\bar{\boldsymbol{R}}^{-1}(i)\bar{\boldsymbol{a}}(\theta_k)},$$

where
$\bar{\boldsymbol{R}}(i) = \boldsymbol{S}_D^H(i)E[\boldsymbol{r}(i)\boldsymbol{r}^H(i)]\boldsymbol{S}_D(i) = E[\bar{\boldsymbol{r}}(i)\bar{\boldsymbol{r}}^H(i)]$,
$\bar{\boldsymbol{a}}(\theta_k) = \boldsymbol{S}_D^H(i)\boldsymbol{a}(\theta_k)$.

– The associated MV is

$$\text{MV} = \frac{1}{\bar{\boldsymbol{a}}^H(\theta_k)\bar{\boldsymbol{R}}^{-1}(i)\bar{\boldsymbol{a}}(\theta_k)}$$

MVDR Design of JIO Scheme

- The expressions of the beamformers $\bar{\boldsymbol{w}}(i)$ and $\boldsymbol{S}_D(i)$ are not closed-form solutions.
- They are functions of each other. Therefore, it is necessary to iterate the expressions with initial values to obtain a solution.
- Existence of multiple solutions (which are identical with respect to the MMSE and symmetrical).
- Global convergence to the optimal reduced-rank LCMV filter (eigen-decomposition with known covariance matrix) has been established.
- The key strategy lies in the joint optimization of the filters.
- The rank D must be adjusted by the designer to ensure appropriate performance or can be estimated via another algorithm.

Adaptive MVDR-LMS Algorithm

Initialize all parameter vectors, dimensions

For each data vector $i = 1, \ldots, Q$ do :

– Perform dimensionality reduction :

$$\bar{\boldsymbol{r}}(i) = \boldsymbol{S}_D^H(i)\boldsymbol{r}(i)$$

– Estimate parameters

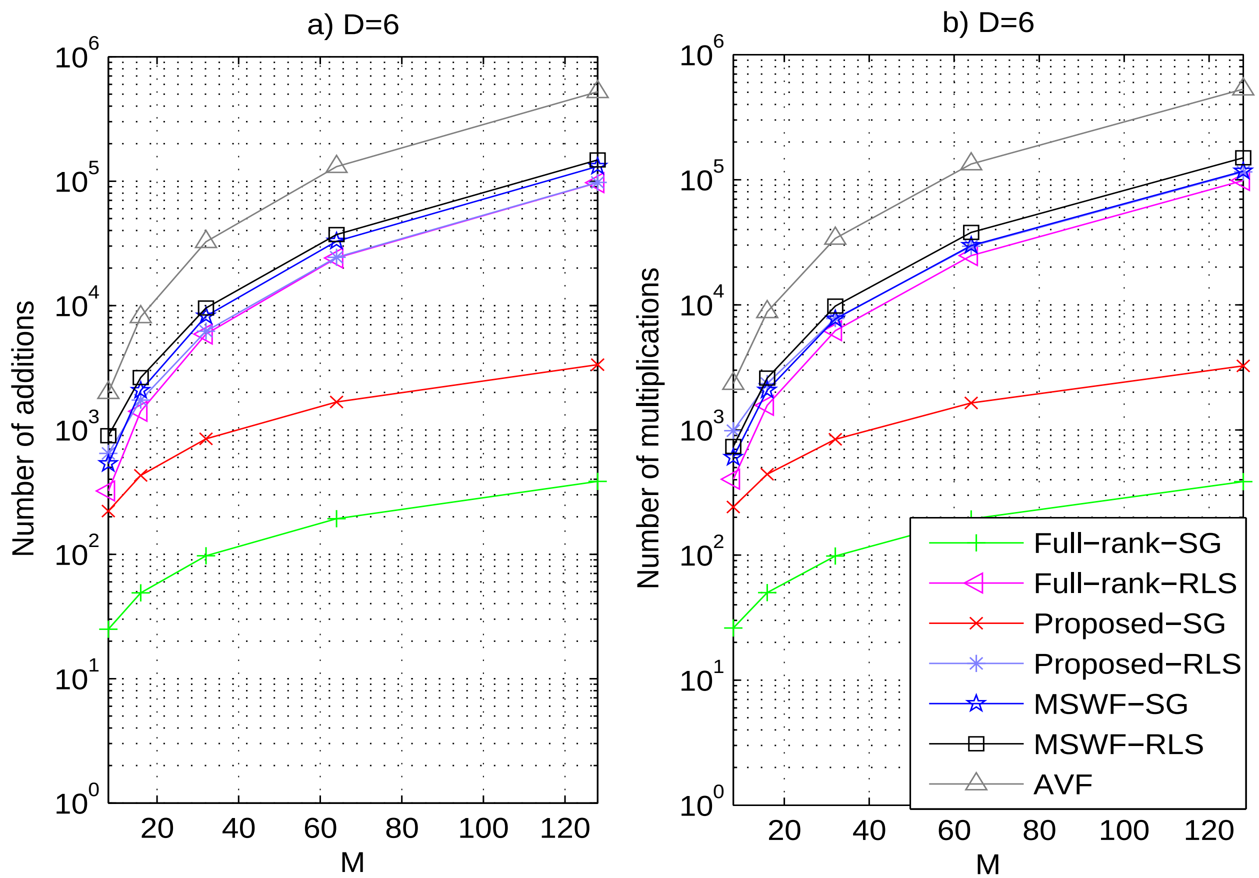
$$\boldsymbol{S}_D(i+1) = \boldsymbol{S}_D(i) - \mu_s x^*(i)\Big[\boldsymbol{r}(i)\bar{\boldsymbol{w}}^H(i) - \boldsymbol{a}(\theta_k)\bar{\boldsymbol{w}}^H(i)\boldsymbol{a}^H(\theta_k)\boldsymbol{r}(i)\Big]$$

$$\bar{\boldsymbol{w}}(i+1) = \bar{\boldsymbol{w}}(i) - \mu_w x^*(i)\Big[\boldsymbol{I} - \Big(\bar{\boldsymbol{a}}^H(\theta_k)\bar{\boldsymbol{a}}(\theta_k)\Big)^{-1}\bar{\boldsymbol{a}}(\theta_k)\bar{\boldsymbol{a}}^H(\theta_k)\Big]\bar{\boldsymbol{r}}(i)$$

Complexity of MVDR-JIO ALgorithms

Algorithm	Additions	Multiplications
Full-rank-SG [1]	$3M+1$	$3M+2$
Full-rank-RLS [1]	$3M^2-2M+3$	$6M^2+2M+2$
JIO-LMS	$3DM+2M$ $+2D-2$	$3DM+M$ $+5D+2$
JIO-RLS	$3M^2-2M+3$ $+3D^2-8D+3$	$7M^2+2M$ $+7D^2+9D$
MSWF-SG [12]	DM^2-M^2 $+3D-2$	DM^2-M^2 $+2DM+4D+1$
MSWF-RLS [12]	$DM^2+M^2+6D^2$ $-8D+2$	DM^2+M^2 $+2DM+3D+2$
AVF [24]	$D((M)^2+3(M-1)^2)-1$ $+D(5(M-1)+1)+2M$	$D(4M^2+4M+1)$ $+4M+2$

Complexity of JIO-MVDR Algorithms



Applications : MVDR Beamforming

– A smart antenna system with a ULA containing M sensor elements and half wavelength inter-element spacing is considered.

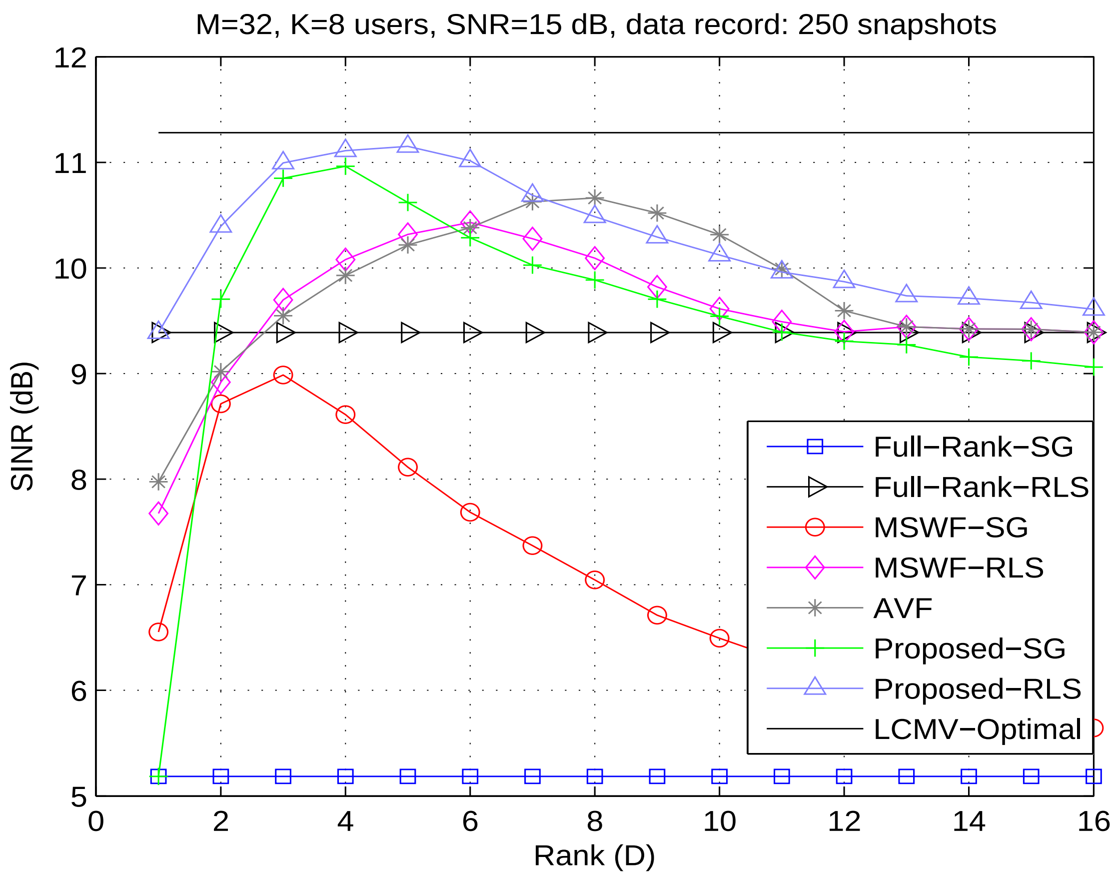
– Figure of merit : the SINR, which is defined as

$$\text{SINR}(i) = \frac{\bar{\boldsymbol{w}}^H(i)\boldsymbol{S}_D^H(i)\boldsymbol{R}_s(i)\boldsymbol{S}_D(i)\bar{\boldsymbol{w}}(i)}{\bar{\boldsymbol{w}}^H(i)\boldsymbol{S}_D^H(i)\boldsymbol{R}_I(i)\boldsymbol{S}_D(i)\bar{\boldsymbol{w}}(i)}$$

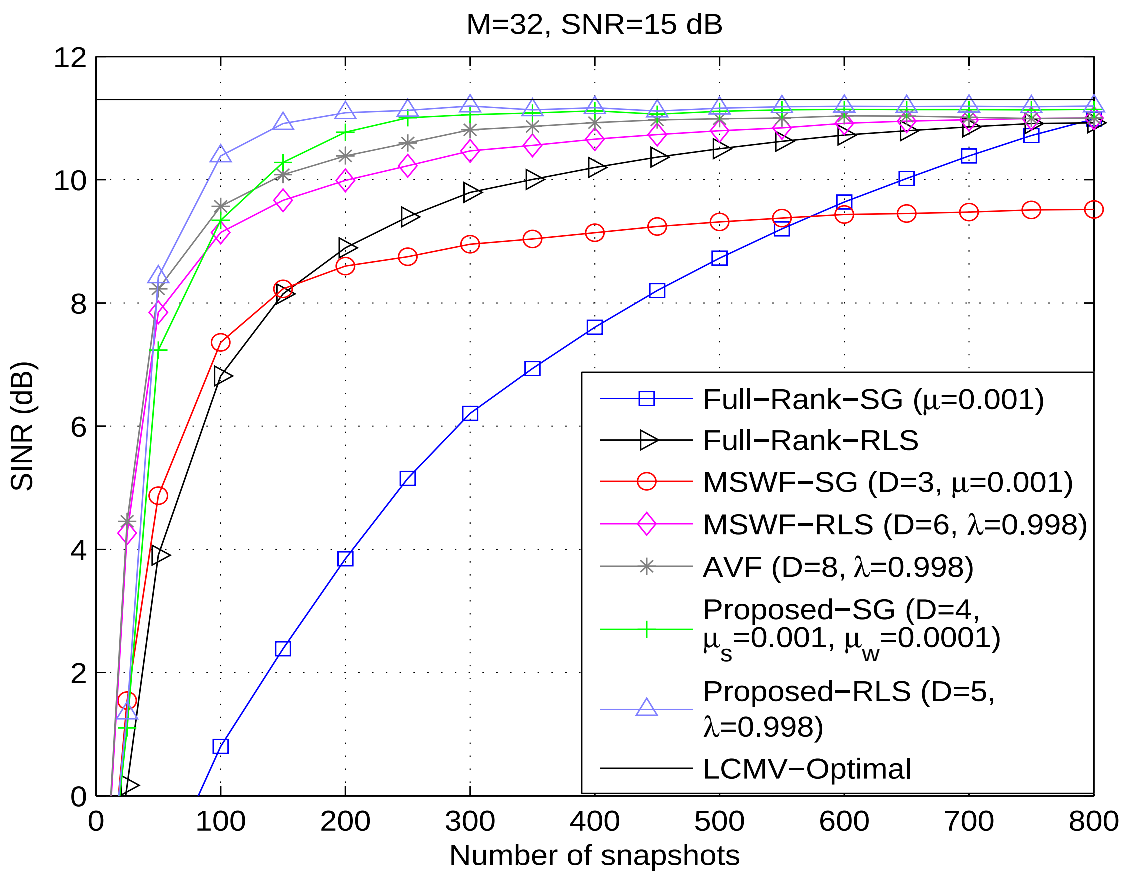
– The signal-to-noise ratio (SNR) is defined as $\text{SNR} = \frac{\sigma_d^2}{\sigma^2}$.

– Initialization : $\bar{\boldsymbol{w}}(0) = [1\ 0\ \ldots\ 0]$ and $\boldsymbol{S}_D(0) = [\boldsymbol{I}_D^T\ \boldsymbol{0}_{D\times(M-D)}^T]$, where $\boldsymbol{0}_{D\times M-D}$ is a $D \times (M-D)$ matrix with zeros in all experiments.

Applications : MVDR Beamforming



Applications : MVDR Beamforming

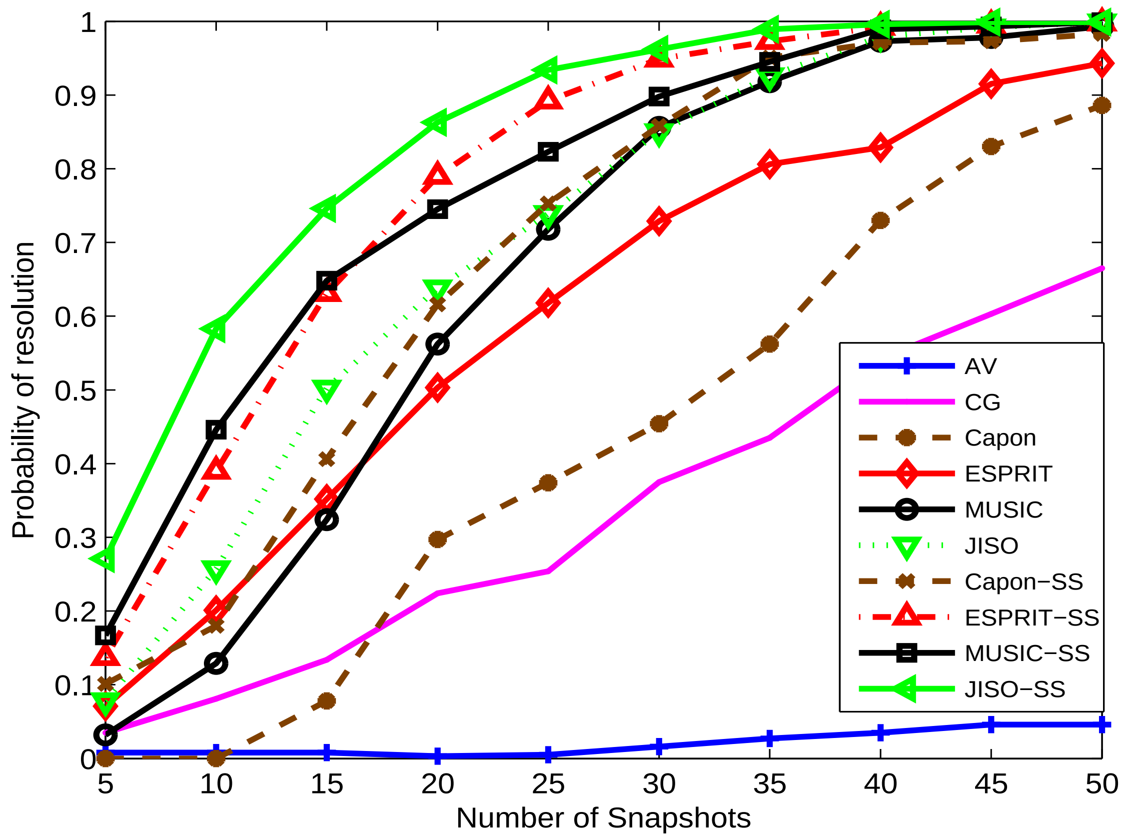


Applications : Direction of Arrival Estimation

- A smart antenna system with a ULA containing M sensor elements and half wavelength inter-element spacing is considered.
- We compare the proposed LCMV JIO method with an LS algorithm with the Capon, MUSIC, ESPRIT, AVF, and CG methods, and run $K = 1000$ iterations to get each curve.
- The spatial smoothing (SS) technique is employed for each algorithm to improve the performance in the presence of correlated sources.
- The DOAs are considered to be resolved if $|\widehat{\theta}_{\text{JISO}} - \theta_k| < 1^o$.
- The probability of resolution is used as a figure of merit and plotted against the number of snapshots.

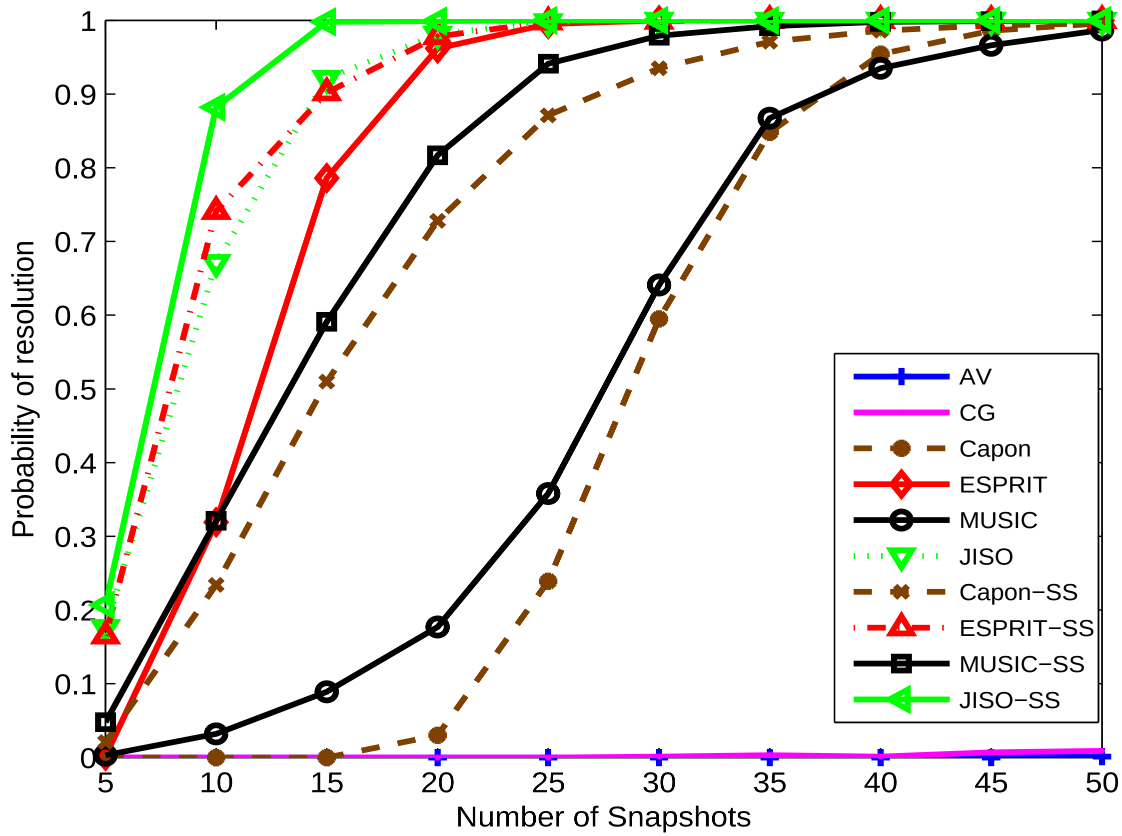
Applications : Direction of Arrival Estimation

Parameters : Probability of resolution versus number of snapshots (separation 3^o, SNR$= -2$dB, $q = 2$, c$= 0.9$, $m = 30$, $r = 6$, $\delta = 5 \times 10^{-4}$, $\alpha = 0.998$, $n = 26$)



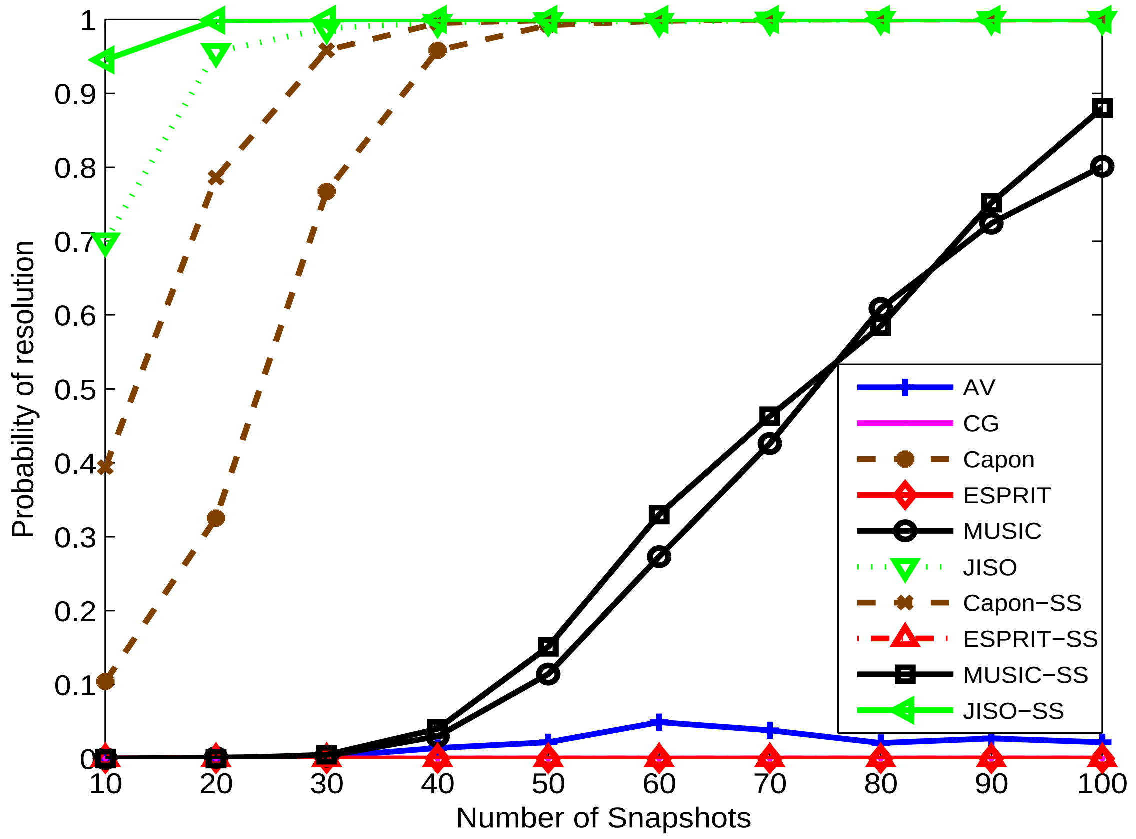
Applications : Direction of Arrival Estimation

Parameters : Probability of resolution versus number of snapshots (separation 3^o, SNR$= -5$dB, q$= 10$, $m = 50$, $r = 6$, $\delta = 5 \times 10^{-4}$, $\alpha = 0.998$, $n = 41$)

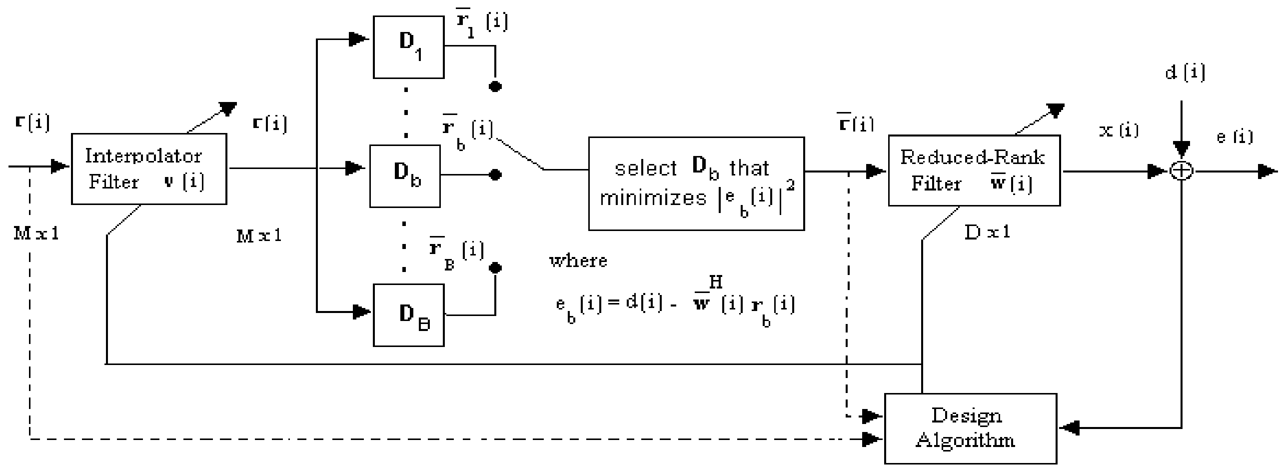


Applications : Direction of Arrival Estimation

Parameters : Probability of resolution versus snapshots (separation 3^o, SNR= 0dB, $q_w = 9$, $m = 50$, $r = 6$, $\delta = 5 \times 10^{-4}$, $\alpha = 0.998$, $n = 41$). We assume an incorrect number of sources $q_w = 9$ instead of $q = 10$.



JIDF Techniques



- Interpolated received vector : $\boldsymbol{r}_I(i) = \boldsymbol{V}^H(i)\boldsymbol{r}(i)$
- Decimated received vector for branch b : $\bar{\boldsymbol{r}}(i) = \boldsymbol{D}_b(i)\boldsymbol{V}^H(i)\boldsymbol{r}(i)$
- Selection of decimation branch $\boldsymbol{D}(i)$: Euclidean distance
- Expression of estimate as a function of $\boldsymbol{v}(i)$, $\boldsymbol{D}(i)$ and $\boldsymbol{w}(i)$:

$$x(i) = \bar{\boldsymbol{w}}^H(i)\boldsymbol{S}_D^H(i)\boldsymbol{r}(i) = \bar{\boldsymbol{w}}^H(i)\boldsymbol{D}_b(i)\boldsymbol{V}^H(i)\boldsymbol{r}(i) = \bar{\boldsymbol{w}}^H(i)\boldsymbol{D}(i)\Re_o(i$$

- Joint optimisation of $\boldsymbol{v}(i)$, $\boldsymbol{D}(i)$ and $\bar{\boldsymbol{w}}(i)$

JIDF Techniques

- Decimation schemes : Optimal, uniform, random, pre-stored.
- The decimation pattern $\boldsymbol{D}(i)$ is selected according to :

$$\boldsymbol{D}(i) = \boldsymbol{D}_b \quad \text{when} \quad \boldsymbol{D}_b(i) = \arg \min_{1 \leq b \leq B} |e_b(i)|^2$$

- Optimal decimator : combinatorial problem with B possibilities

$$\boldsymbol{B} = \underbrace{M \cdot (M-1) \ldots (M - M/L + 1)}_{M/L \ \text{terms}} = \frac{M!}{(M - M/L)!}$$

- Suboptimal decimation schemes :

  - Uniform (U) Decimation

  - Pre-Stored (PS) Decimation.

  - Random (R) Decimation.

JIDF Techniques

– General framework for decimation schemes

$$
\boldsymbol{D}_b = \begin{bmatrix}
\underbrace{0 \;\ldots\; 0}_{r_1\ zeros} & 1 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\underbrace{0 \;\; 0 \;\ldots\; 0}_{r_m\ zeros} & 1 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\underbrace{0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \ldots 0}_{r_D\ zeros} & 1 & \underbrace{0 \;\ldots\; 0}_{(M-r_D-1)\ zeros} & & & & & & &
\end{bmatrix}
$$

where m $(m = 1, 2, \ldots, M/L)$ denotes the m-th row and r_m is the number of zeros given by the decimation strategy.

– Suboptimal decimation schemes :

a. Uniform (U) Decimation with $B = 1 \rightarrow r_m = (m-1)L$.

b. Pre-Stored (PS) Decimation. We select $r_m = (m-1)L + (b-1)$ which corresponds to the utilization of uniform decimation for each branch b out of B branches.

c. Random (R) Decimation. We choose r_m as a discrete uniform random variable between 0 and $M-1$.

Linear MMSE Design of Parameter Vectors

– The MMSE expressions for $\bar{\mathbf{w}}(i)$ and $\mathbf{v}(i)$ can be computed via the minimization of the cost function

$$J_{\text{MSE}}^{(\mathbf{v}(i),\mathbf{D}(i),\bar{\mathbf{w}}(i))} = E[|d(i) - \mathbf{v}^H(i)\boldsymbol{\Re}_o^T(i)\mathbf{D}^T(i)\bar{\mathbf{w}}^*(i)|^2]$$

– Fixing the interpolator $\mathbf{v}(i)$ and minimizing the cost function with respect to $\bar{\mathbf{w}}(i)$ the interpolated Wiener filter weight vector is

$$\bar{\mathbf{w}}(i) = \boldsymbol{\alpha}(\mathbf{v}) = \bar{\mathbf{R}}^{-1}(i)\bar{\mathbf{p}}(i)$$

where
$\bar{\mathbf{R}}(i) = E[\bar{\mathbf{r}}(i)\bar{\mathbf{r}}^H(i)]$,
$\bar{\mathbf{p}}(i) = E[d^*(i)\bar{\mathbf{r}}(i)]$,
$\bar{\mathbf{r}}(i) = \boldsymbol{\Re}(i)\mathbf{v}^*(i)$.

Linear MMSE Design of Parameter Vectors

– Fixing $\bar{\mathbf{w}}(i)$ and minimizing the cost function with respect to $\mathbf{v}(i)$ the interpolator weight vector is

$$\mathbf{v}(i) = \boldsymbol{\beta}(\bar{\mathbf{w}}) = \mathbf{R}_u^{-1}(i)\mathbf{p}_u(i)$$

where
$\mathbf{R}_u(i) = E[\mathbf{u}(i)\mathbf{u}^H(i)]$, $\mathbf{p}_u(i) = E[d^*(i)\mathbf{u}(i)]$ and $\mathbf{u}(i) = \Re^T(i)\bar{\mathbf{w}}^*(i)$

– The associated MSE expressions are

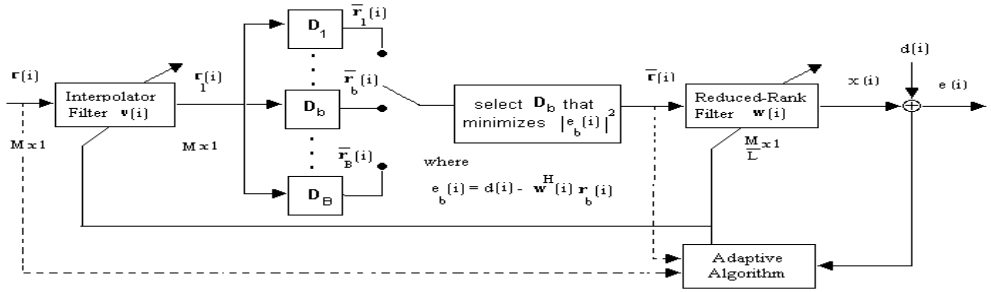
$$J(\mathbf{v}) = J_{\text{MSE}}(\boldsymbol{\alpha}(\mathbf{v}), \mathbf{v}) = \sigma_d^2 - \bar{\mathbf{p}}^H(i)\bar{\mathbf{R}}^{-1}(i)\bar{\mathbf{p}}(i)$$

$$J_{\text{MSE}}(\bar{\mathbf{w}}, \boldsymbol{\beta}(\bar{\mathbf{w}})) = \sigma_d^2 - \mathbf{p}_u^H(i)\mathbf{R}_u^{-1}(i)\mathbf{p}_u(i)$$

where $\sigma_d^2 = E[|d(i)|^2]$.

– The points of global minimum can be obtained by $\mathbf{v}_{\text{opt}} = \arg\min_{\mathbf{v}} J(\mathbf{v})$ and $\bar{\mathbf{w}}_{\text{opt}} = \boldsymbol{\alpha}(\mathbf{v}_{\text{opt}})$ or $\bar{\mathbf{w}}_{\text{opt}} = \arg\min_{\bar{\mathbf{w}}} J_{\text{MSE}}(\bar{\mathbf{w}}, \boldsymbol{\beta}($ and $\mathbf{v}_{\text{opt}} = \boldsymbol{\beta}(\bar{\mathbf{w}}_{\text{opt}})$.

Adaptive JIDF-LMS Algorithms



Initialize all parameter vectors, dimensions, number of branches B and select decimation technique

For each data vector $i = 1, \ldots, Q$ do :

– Select decimation branch that minimizes $e_b(i) = d(i) - \boldsymbol{w}^H(i)\bar{\boldsymbol{r}}(i)$
– Make $\bar{\boldsymbol{r}}(i) = \bar{\boldsymbol{r}}_b(i)$ when $b = \arg\min_{1 \le b \le B} |e_b(i)|^2$
– Estimate parameters

$$\boldsymbol{v}(i+1) = \boldsymbol{v}(i) + \eta e^*(i)\boldsymbol{u}(i)$$

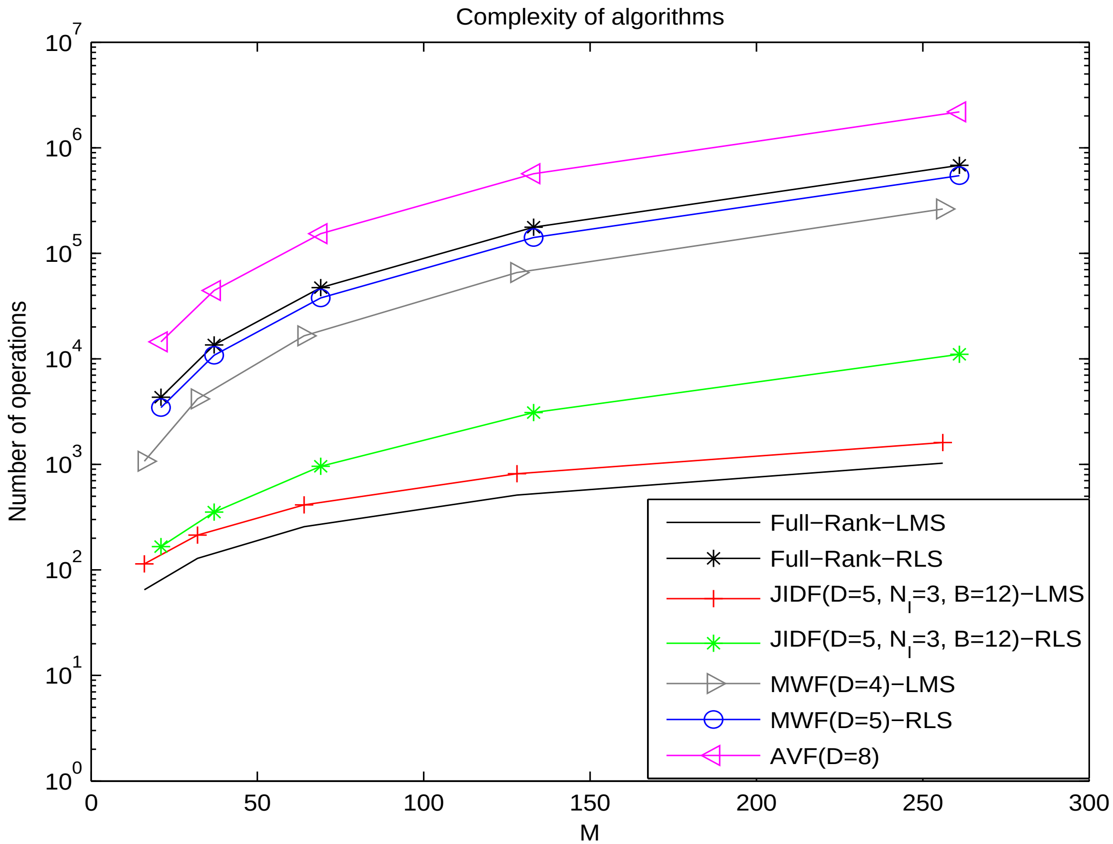
$$\bar{\boldsymbol{w}}(i+1) = \bar{\boldsymbol{w}}(i) + \mu e^*(i)\bar{\boldsymbol{r}}(i)$$

where $\boldsymbol{u}(i) = \Re^T(i)\bar{\boldsymbol{w}}^*(i)$ and $\bar{\boldsymbol{r}}(i) = \boldsymbol{D}(i)\boldsymbol{V}^H(i)\boldsymbol{r}(i)$.

Complexity of JIDF Algorithms

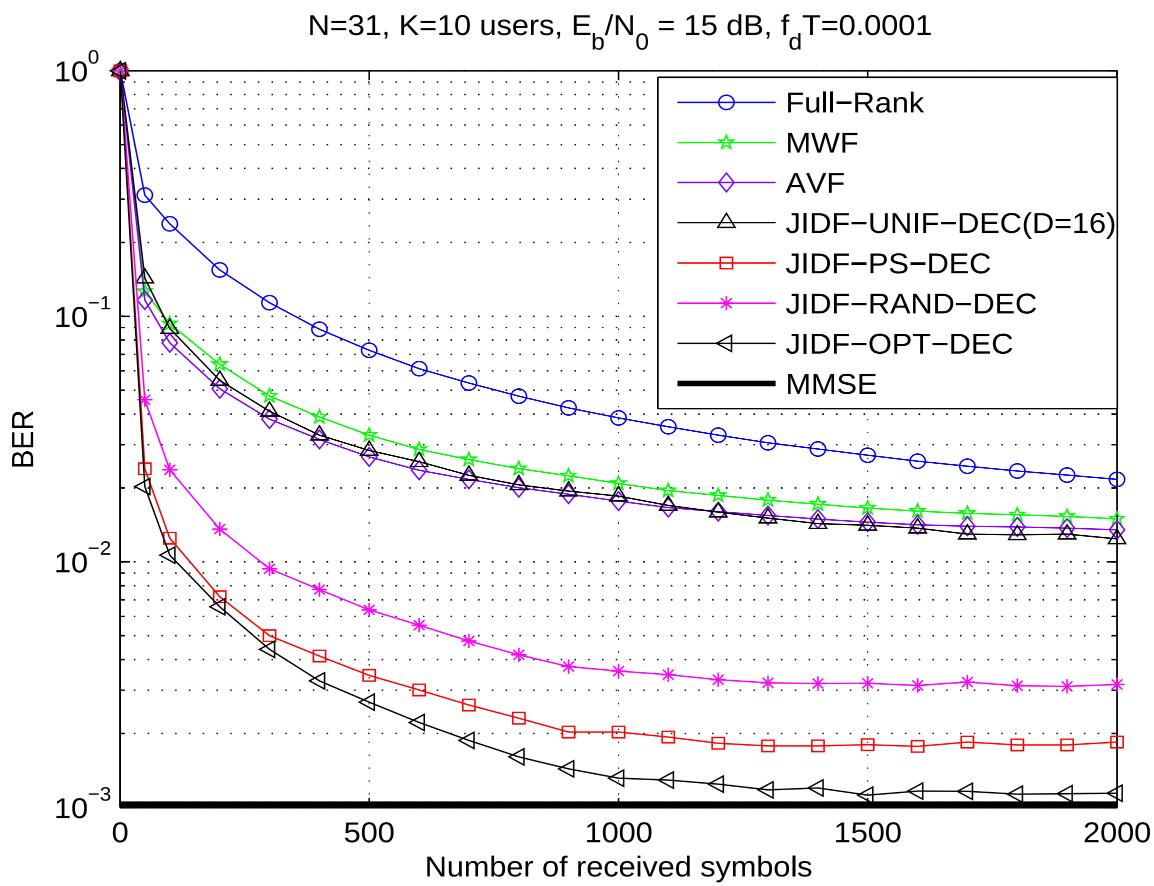
	Number of operations per symbol	
Algorithm	Additions	Multiplications
Full-rank-LMS	$2M$	$2M+1$
Full-rank-RLS	$3(M-1)^2+M^2+2M$	$6M^2+2M+2$
JIDF-LMS	$(B+1)(D)+2N_I$	$(B+2)D$
JIDF-RLS	$3(D-1)^2+3(N_I-1)^2$ $+(D-1)N_I+N_IM+(D)^2$ $+N_I^2+(B+1)D+2N_I$	$6(D)^2+6N_I^2$ $+DN_I+2$ $+(B+2)D+N_I$
MWF-LMS	$D(2(\bar{M}-1)^2+\bar{M}+3)$	$D(2\bar{M}^2+5\bar{M}+7)$
MWF-RLS	$D(4(\bar{M}-1)^2+2\bar{M})$	$D(4\bar{M}^2+2\bar{M}+3)$
AVF	$D((M)^2+3(M-1)^2)-1$ $+D(5(M-1)+1)+2M$	$D(4(M)^2+4M+1)$ $+4M+2$

Complexity of JIDF Algorithms



Applications : Interference suppression for CDMA

Parameters : Uplink scenario, QPSK symbols, K users, N chips per symbol and L propagation paths, receiver filter has $M = N + L_p - 1$ taps.



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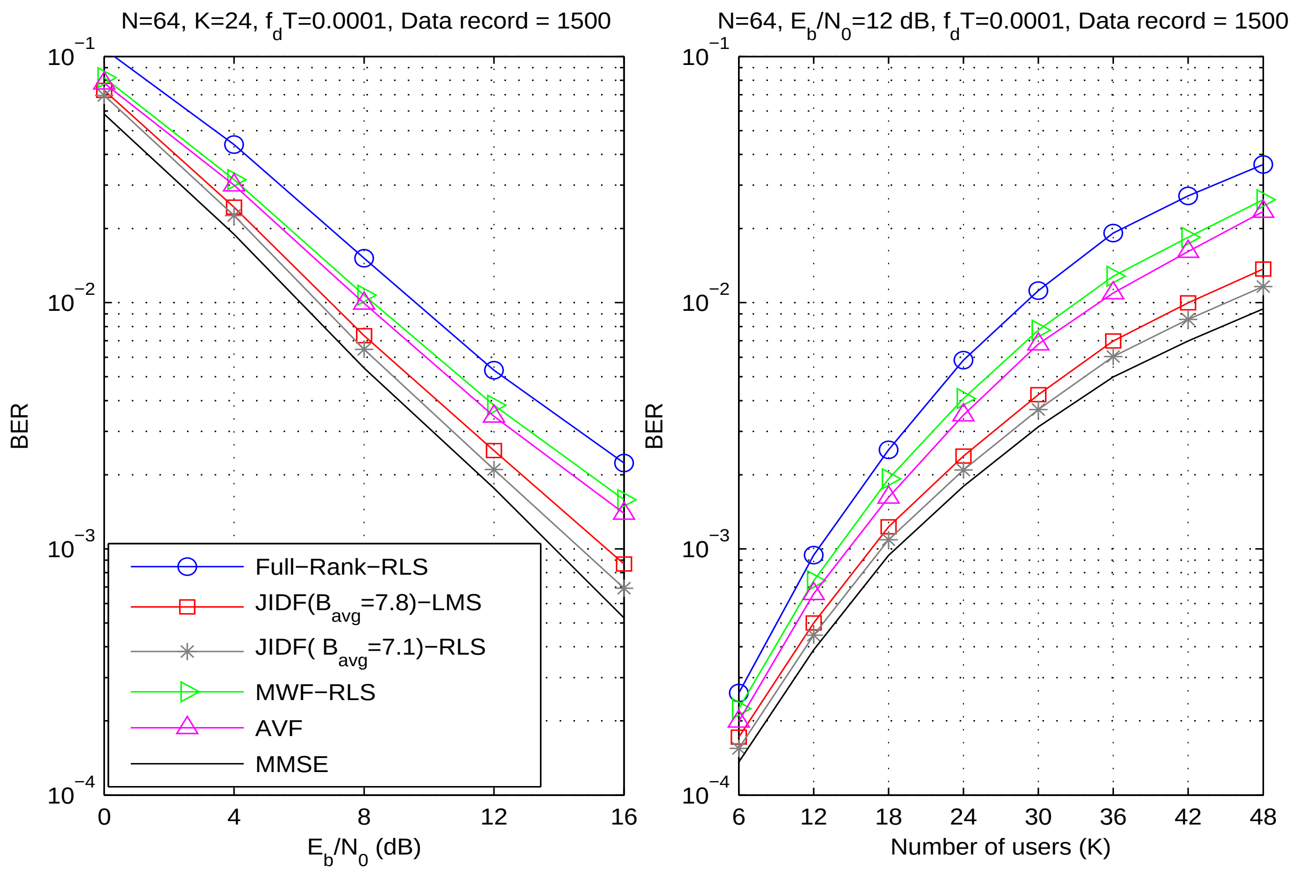
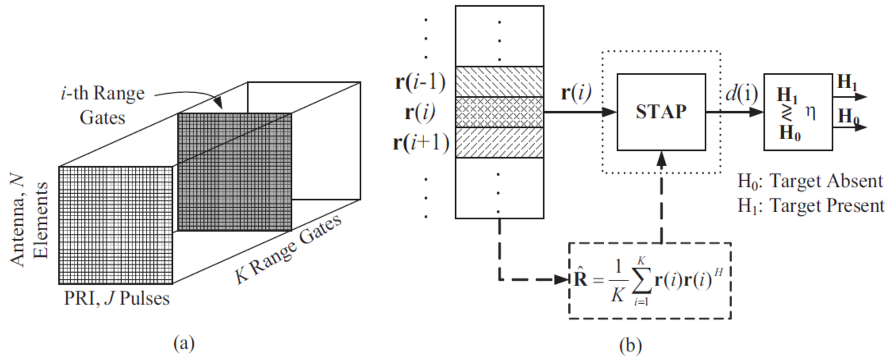


Applications : STAP for Radar Systems



- The system under consideration is a pulsed Doppler radar residing on an airborne platform.
- The radar antenna is a uniformly spaced linear antenna array consisting of N elements. The radar returns are collected in a coherent processing interval (CPI).
- The $M \times 1$ radar space-time snapshot $\boldsymbol{r}(i)$ is then expressed for each of the two hypotheses in the following form

$$H_0 : \boldsymbol{r}(i) = \boldsymbol{v}(i);$$
$$H1 : \boldsymbol{r}(i) = a\boldsymbol{s} + \boldsymbol{v}(i);$$

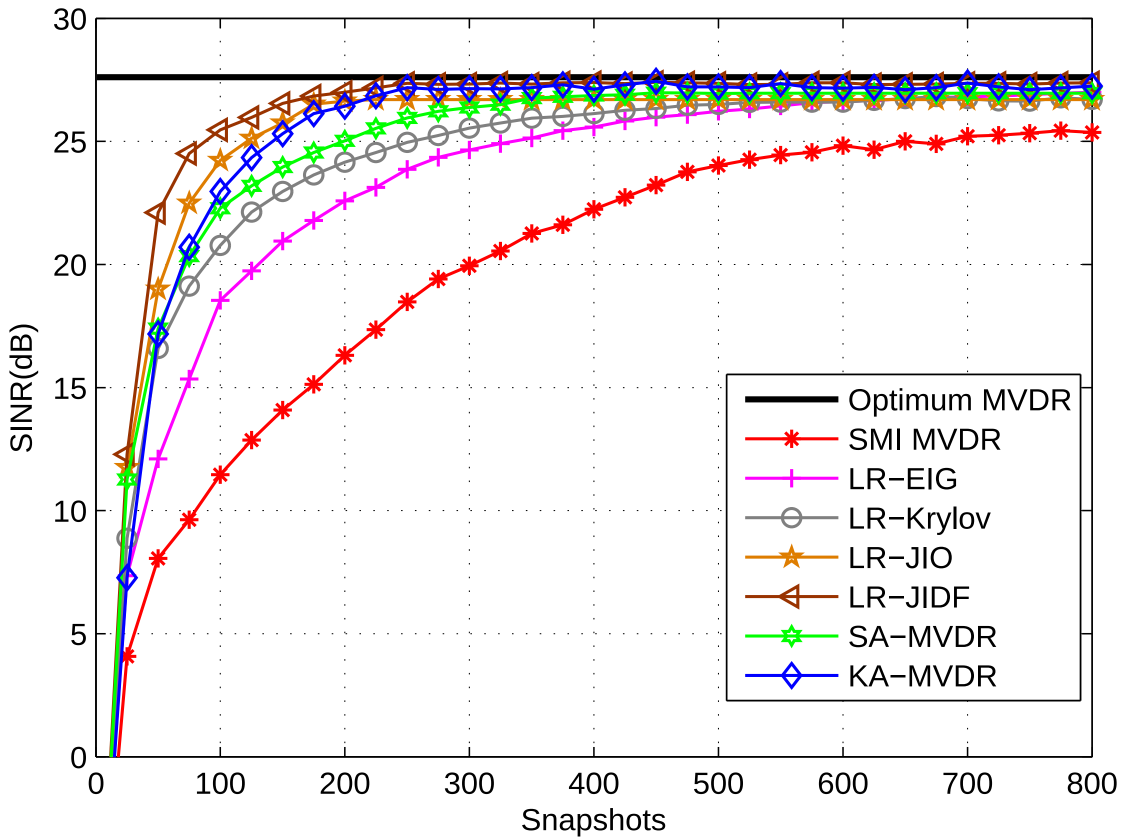
- Problem : to design a spatial-temporal beamformer with limited training.

Applications : STAP for Radar Systems

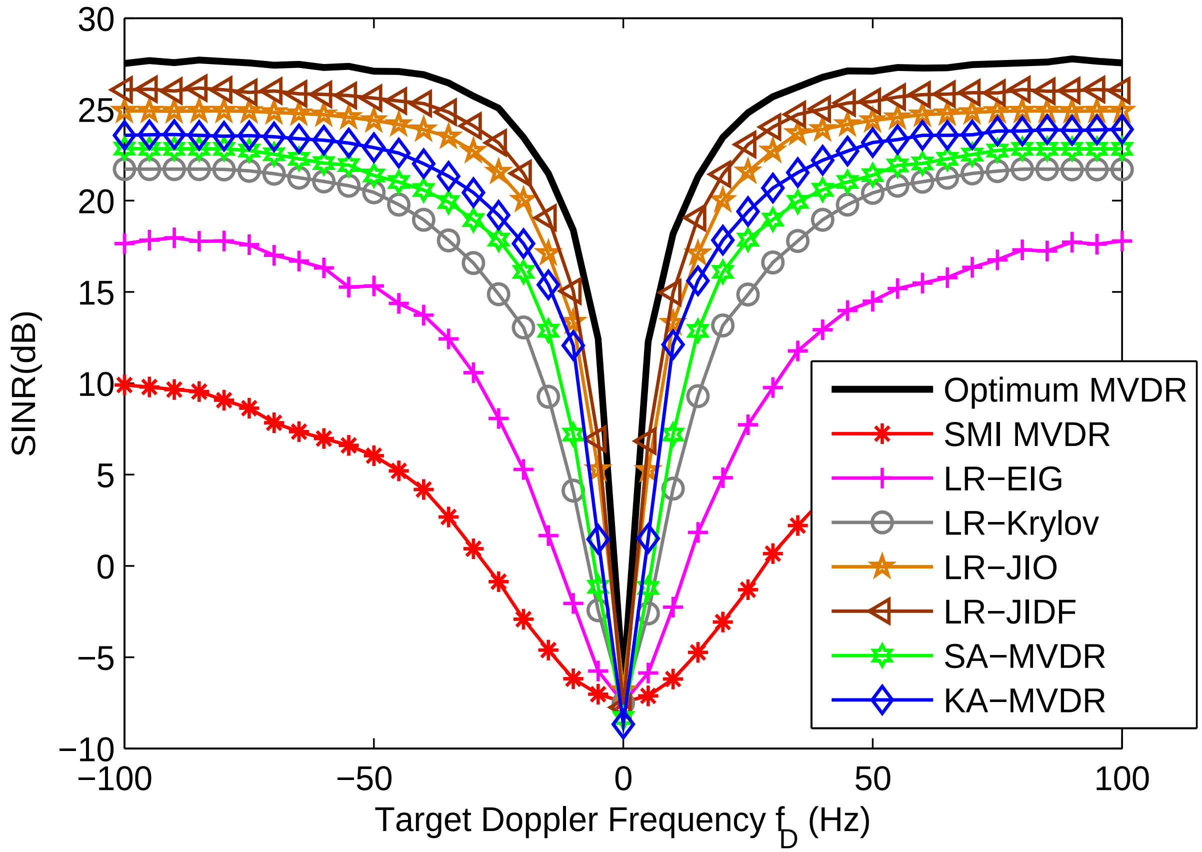
Airborne radar system parameters :

Parameter	Value
Antenna array	Sideway-looking array (SLA)
Carrier frequency (f_c)	450 MHz
Transmit pattern	Uniform
PRF (f_r)	300 Hz
Platform velocity (v)	75 m/s
Platform height (h)	9000 m
Clutter-to-Noise ratio (CNR)	40 dB
Elements of sensors (N)	8
Number of Pulses (J)	8

Applications : STAP for Radar Systems



Applications : STAP for Radar Systems

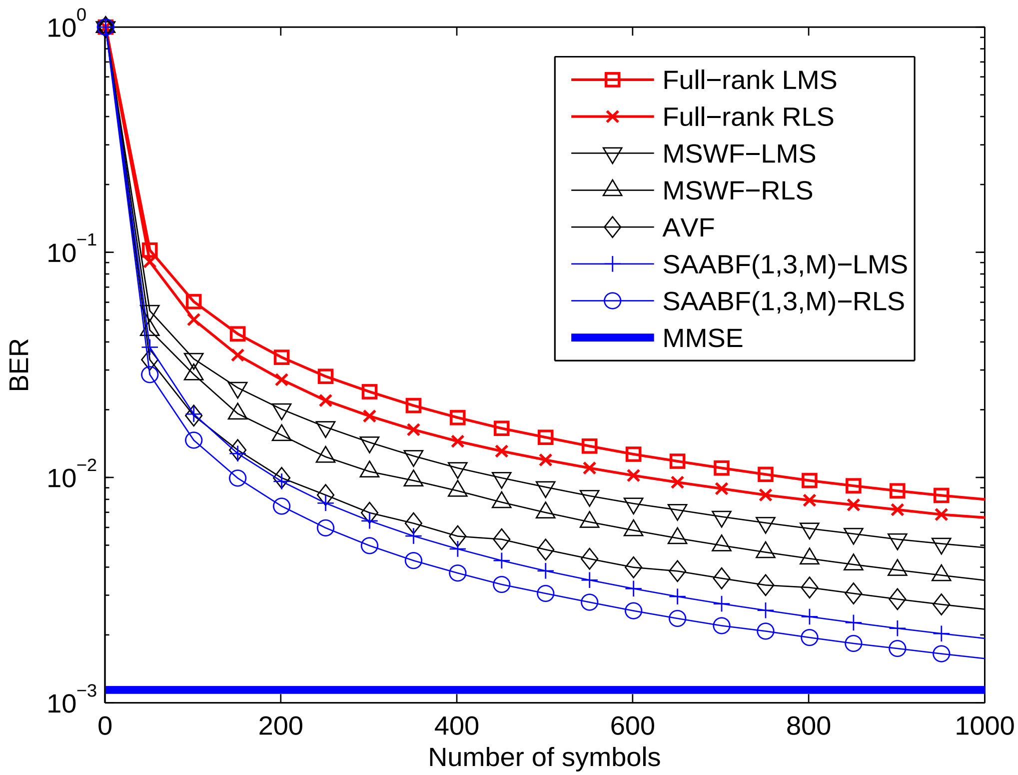


Applications : UWB communications

- We apply the a variation of the JIDF scheme called SAABF to the downlink of a multiuser BPSK DS-UWB system and evaluate their performance against existing methods.
- In all numerical simulations, the pulse shape adopted is the RRC pulse with the pulse-width 0.375ns.
- The spreading codes are generated randomly with a spreading gain of 24 and the data rate of the communication is approximately 110Mbps.
- The standard IEEE 802.15.4a channel model for the NLOS indoor environment is employed.
- We assume that the channel is constant during the whole transmission.
- The sampling rate at the receiver is assumed to be 8GHz that is the same as the standard channel model and the observation window length M for each data symbol is set to 120 samples.

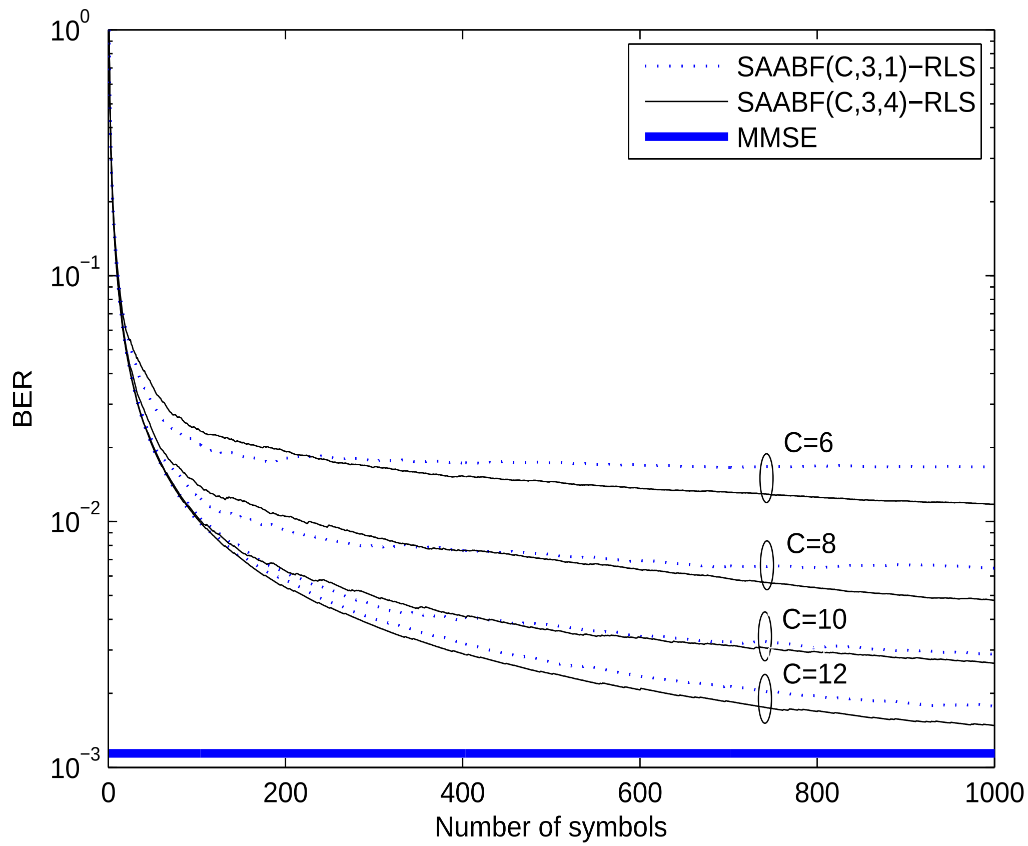
Applications : UWB communications

Parameters :BER performance of different algorithms for a SNR=16dB and 3 users. The following parameters were used : full-rank LMS ($\mu = 0.075$), full-rank RLS ($\lambda = 0.998$, $\delta = 10$), MSWF-LMS ($D = 6$, $\mu = 0.075$), MSWF-RLS ($D = 6$, $\lambda = 0.998$), AVF ($D = 6$), SAABF (1,3,M)-LMS ($\mu_w = 0.1$, $\mu_\psi = 0.2$, 2 iterations) and SAABF (1,3,M)-RLS ($\lambda = 0.998$, $\delta = 0.1$, 1 iteration).

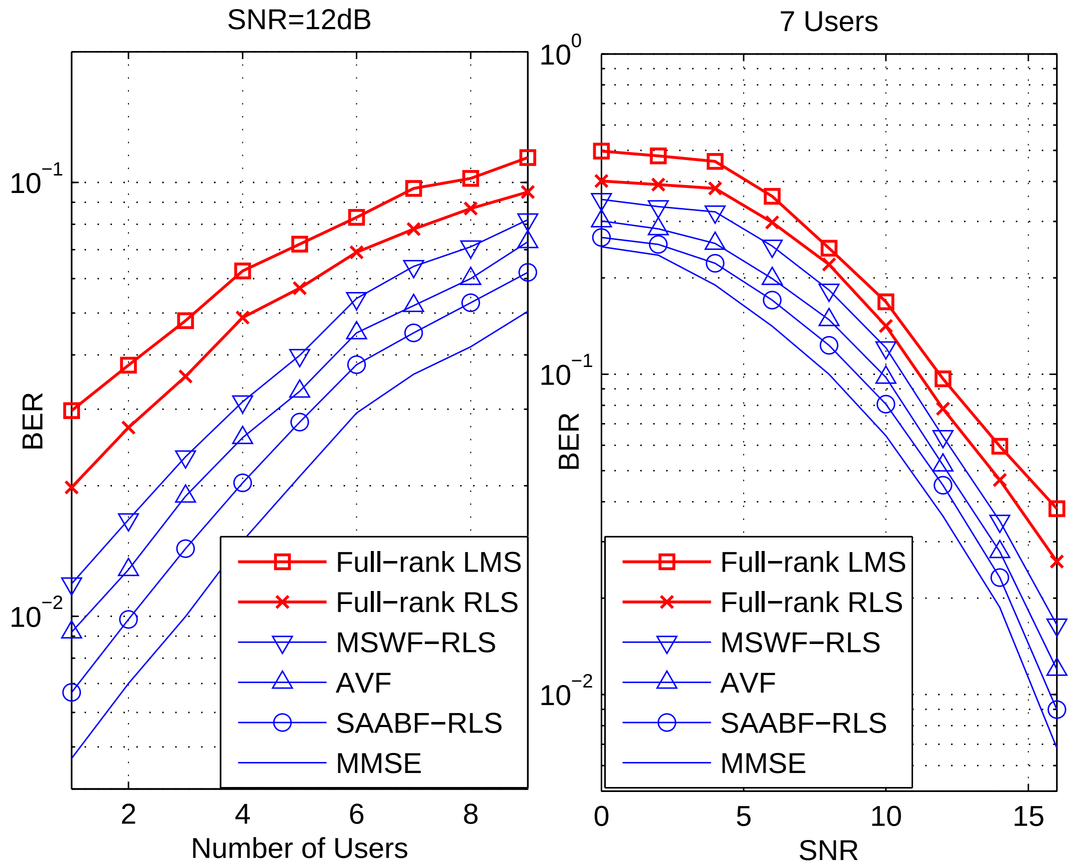


Applications : UWB communications

Parameters :BER performance of the proposed SAABF scheme versus the number of training symbols for a SNR=16dB. The number of users is 3 and the following parameters were used : SAABF-RLS ($\lambda = 0.98$, $\delta = 10$).



Applications : UWB communications



Model-order selection techniques

- Basic principle : to determine the best fit between observed data and the model used.
- General approaches to model-order selection :
  - Setting of upper bounds on models with "some" prior knowledge : one of the most used in communications.
  - Akaike's information theoretic criterion : works well, requires a large number of computations, not suitable to time-varying scenarios.
  - Minimum description length (MDL) : also works well, not suitable to time-varying scenarios.
  - Adaptive filtering approach : use for adaptive algorithms with dynamic lengths, work well and have lower complexity than prior art.

Akaike Information Criterion

– Basic principle : employs information entropy to perform model order selection.
– Method :

$$\text{AIC} = 2k - 2l(\widehat{\theta}),$$

where

k is the number of parameters

$\widehat{\theta}$ is the ML estimate

$l(\cdot)$ is the log likelihood function.

– The lower the AIC the better the model selected.
– It is more suitable to ML estimation problems.

Minimum Description Length

- Basic principle : given a data set and competing statistical models, the best one is that which provides the shortest description length.
- Method :

$$\text{MDL} = 1/2 m ln N - l(\widehat{\theta}),$$

  where

  N is the number of samples

  m is the number of independently adjusted parameters

  $l(\cdot)$ is the log likelihood function.
- The shortest the MDL the better the model selected.
- It is more suitable to ML estimation problems.
- The MDL converges to the true model order.

Model-Order Selection for Time-Varying Scenarios

- Approaches used for reduced-rank techniques :
  - Testing of orthogonality conditions between columns of transformation matrix $\boldsymbol{S}_D(i)$ [12] :
    - used with the MSWF for selecting the rank D.
  - Cross-validation of data [24] :
    - used with the AVF,
    - works well but can be complex since the algorithms sometimes selects D quite large.
    - This can be a problem if M is large and D approaches it.
  - Use of a priori values of least-squares type cost functions with lower and upper bounds :
    - works very well and it is simple to use and design [12, 17, 35].
    - It can be easily extended when the designer has multiple parameters with orders to adjust.

Model-order selection with the JIO-MVDR algorithm

- Consider the exponentially weighted *a posteriori* least-squares type cost function described by

$$\mathcal{C}(\boldsymbol{S}_D(i-1), \bar{\boldsymbol{w}}^{(D)}(i-1)) = \sum_{l=1}^{i} \alpha^{i-l} \left| \bar{\boldsymbol{w}}^{H,\ (D)}(i-1) \boldsymbol{S}_D(i-1) \boldsymbol{r}(l) \right|^2,$$

  where α is the forgetting factor and $\bar{\mathbf{w}}^{(D)}(i-1)$ is the reduced-rank filter with rank D.
- For each time interval i, we can select the rank D_{opt} which minimizes $\mathcal{C}(\boldsymbol{S}_D(i-1), \bar{\boldsymbol{w}}^{(D)}(i-1))$ and the exponential weighting factor α is required as the optimal rank varies as a function of the data record.
- The key quantities to be updated are $\boldsymbol{S}_D(i)$,$\bar{\boldsymbol{w}}(i)$, $\bar{\boldsymbol{a}}(\theta_k)$ and $\bar{\boldsymbol{P}}(i)$ (RLS algorithm).

Model-order selection with JIO-MVDR algorithms

– Let us define the following extended matrix $\boldsymbol{S}^{(D)}(i)$ and the extended reduced-rank beamformer $\bar{\boldsymbol{w}}^{(D)}(i)$ as follows :

$$\boldsymbol{S}^{(D)}(i) = \begin{bmatrix} s_{1,1} & \cdots & s_{1,D_{\text{min}}} & \cdots & s_{1,D_{\text{max}}} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ s_{M,1} & \cdots & s_{M,D_{\text{min}}} & \cdots & s_{M,D_{\text{max}}} \end{bmatrix} \tag{1}$$

and

$$\bar{\boldsymbol{w}}^{(D)}(i) = \begin{bmatrix} w_1 \\ \vdots \\ w_{D_{\text{min}}} \\ \vdots \\ w_{D_{\text{max}}} \end{bmatrix}$$

– $\boldsymbol{S}^{(D)}(i)$ and $\bar{\boldsymbol{w}}^{(D)}(i)$ are updated along with the associated quantities $\bar{\boldsymbol{a}}(\theta_k)$ and $\bar{\boldsymbol{P}}(i)$ for the maximum allowed rank D_{max}.

Model-order selection with JIO-MVDR algorithms

– The model-order selection algorithm determines the rank that is best for each time instant i using the cost function.

– The model-order selection algorithm is then given by

$$D_{\mathrm{opt}} = \arg \min_{D_{\mathrm{min}} \leq d \leq D_{\mathrm{max}}} \mathcal{C}(\boldsymbol{S}_D(i-1), \bar{\boldsymbol{w}}^{(D)}(i-1))$$

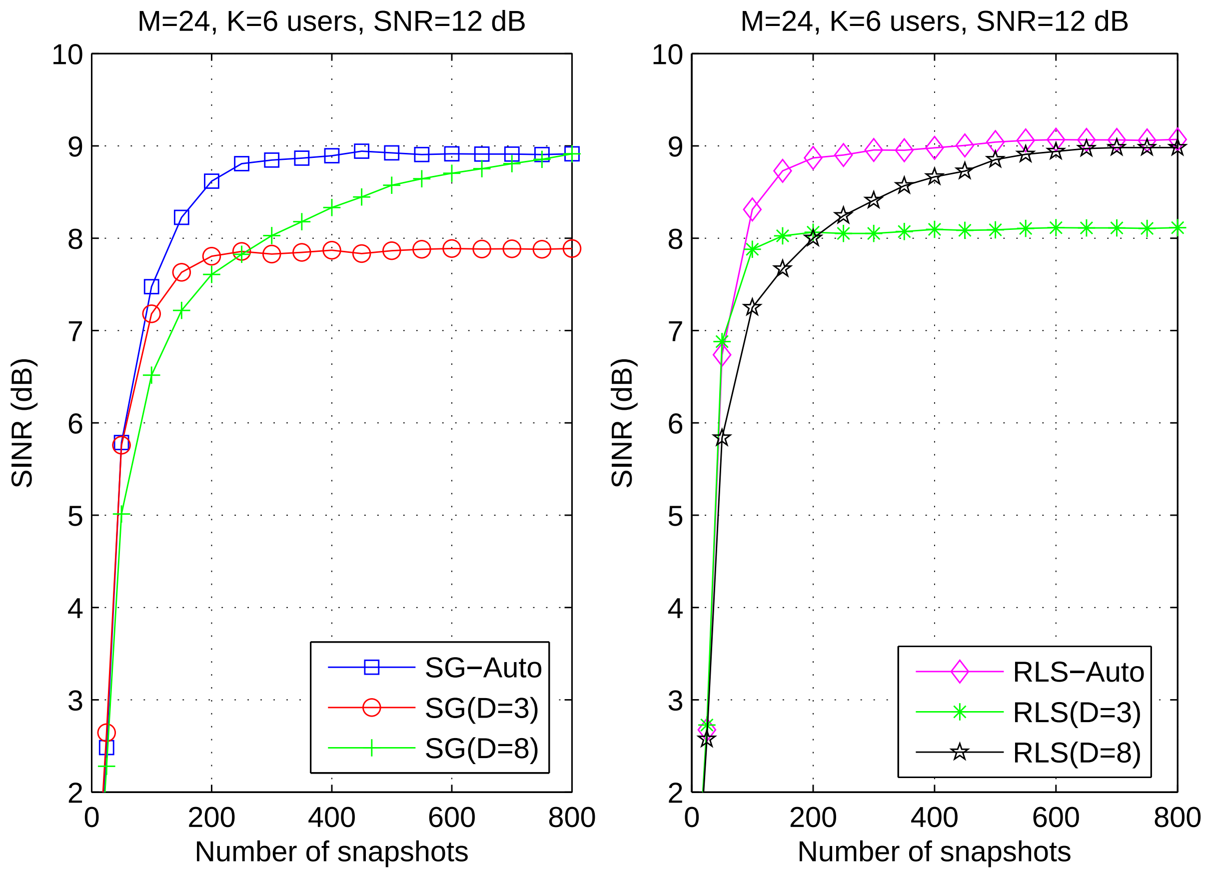
where

d is an integer,

D_{min} and D_{max} are the minimum and maximum ranks allowed for the reduced-rank filter, respectively.

Model-order selection with JIO-MVDR algorithms

SINR performance of LCMV (a) SG and (b) RLS algorithms against snapshots with $M = 24$, $SNR = 12$ dB with automatic rank selection.



Model-order selection with JIDF algorithms

- Consider the following exponentially weighed *a posteriori* least-squares type cost function

$$\mathcal{C}(\bar{\mathbf{w}}^{(D)}, \mathbf{v}^{(N_I)},\ \mathbf{D}) = \sum_{l=1}^{i} \alpha^{i-l} \Big| d(l) - \bar{\mathbf{w}}^{H,\ (D)}(l)\mathbf{D}(l)\Re_o(l)\mathbf{v}^{*,\ (N_I)}(l)\Big|^2$$

  where

  α is the forgetting factor,

  $\tilde{\mathbf{w}}^{(D)}(i-1)$ is the reduced-rank filter with rank D and

  $\mathbf{v}^{(N_I)}(i)$ is the interpolator filter with rank N_I.

- For each time interval i and a given decimation pattern and B, we can select D and N_I which minimizes $\mathcal{C}(\bar{\mathbf{w}}^{(D)}, \mathbf{v}^{(N_I)},\ \mathbf{D})$.

Model-order selection with JIDF algorithms

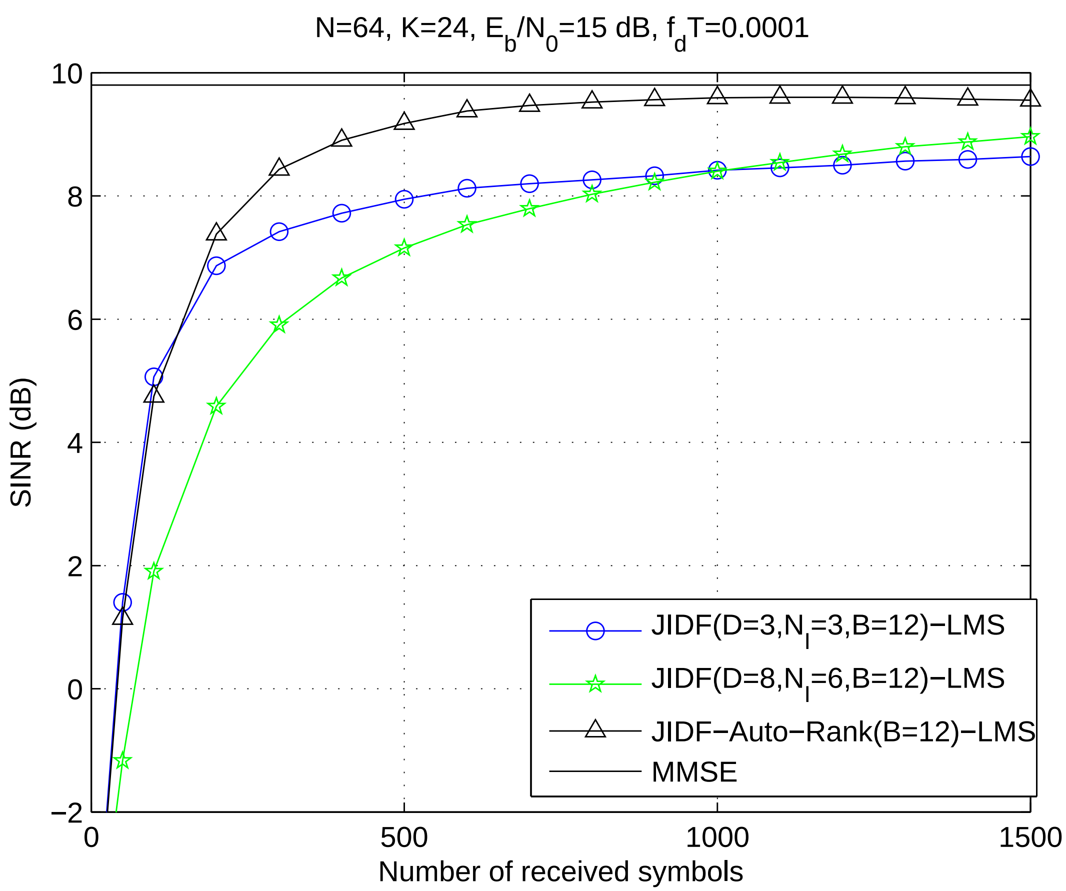
– The model-order selection algorithm that chooses the best lengths D_{opt} and $N_{I_{\text{opt}}}$ for the filters $\mathbf{v}(i)$ and $\bar{\mathbf{w}}(i)$, respectively, is given by

$$\{D_{\text{opt}}, N_{I_{\text{opt}}}\} = \arg \min_{\substack{N_{I_{\text{min}}} \leq n \leq N_{I_{\text{max}}} \\ D_{\text{min}} \leq d \leq D_{\text{max}}}} \mathcal{C}(\bar{\mathbf{w}}^{(d)}, \mathbf{v}^{(n)},\ \mathbf{D})$$

where
d and n are integers,
D_{min} and D_{max} and
$N_{I_{\text{min}}}$ and $N_{I_{\text{max}}}$ are the minimum and maximum ranks allowed for $\bar{\mathbf{w}}(i)$ and $\mathbf{v}(i)$, respectively.

Model-order selection with JIDF algorithm

SINR performance against rank (D) for the analyzed schemes using LMS and RLS algorithms.



Applications, perspectives and future work

- Applications : interference suppression, beamforming, channel estimation, echo cancellation, target tracking, wireless sensor networks, signal compression, radar, control, seismology, etc.

- Perspectives :
  - Work in this field is not fully explored.
  - Many unsolved problems when dimensions become large : estimation, tracking, general acquisition, networks, distributed problems .

- Future work :
  - Information theoretic study of very large observation data : performance limits as M goes to infinity.
  - Development of vector and matrix-based parameter estimates as opposed to current scalar parameter estimation of existing methods.
  - Distributed reduced-rank processing.

Concluding remarks

- Reduced-rank signal processing is a powerful set of tools that allow the processing of large data vectors, enabling a substantial reduction in training and complexity.

- An overview of reduced-rank techniques, detailing eigen-decompo methods and the MSWF, was presented along with applications in communications and sensor array systems.

- A family of reduced-rank algorithms based on JIO techniques was presented along with applications.

- A recently proposed reduced-rank scheme called JIDF was also briefly reviewed with applications.

- Several applications have been considered as well as a number of future investigation topics have been discussed.

Questions ?

Thank you !

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References

1. S. Haykin, *Adaptive Filter Theory*, 4rd edition, Prentice-Hall, Englewood Cliffs, NJ, 2002.

2. H. L. Van Trees, "Detection, Estimation, and Modulation Theory, Part IV, Optimum Array Processing", John Wiley and Sons, 2002.

3. L. L. Scharf, "The SVD and reduced rank signal processing," Signal Processing, vol. 25, no. 2, pp. 113-133, 1991.

4. L. L. Scharf and D. W. Tufts, "Rank reduction for modeling stationary signals," *IEEE Transactions on Acoustics, Speech and Signal Processing*, vol. ASSP-35, pp. 350-355, March 1987.

5. L. L. Scharf and B. van Veen, "Low rank detectors for Gaussian random vectors," *IEEE Transactions on Acoustics, Speech and Signal Processing*, vol. ASSP-35, pp. 1579-1582, November 1987.

6. H. Wang and L. Cai, "On adaptive spatial-temporal processing for airborne surveillance radar systems," IEEE Trans. Aerospace and Electronic Systems, vol. 30, pp. 660-699, Jul 1994.

7. A. M. Haimovich and Y. Bar-Ness, "An eigenanalysis interference canceler," *IEEE Trans. on Signal Processing*, vol. 39, pp. 76-84, Jan. 1991.

8. X. Wang and H. V. Poor, “Blind multiuser detection : A subspace approach,” *IEEE Trans. on Inf. Theory*, vol. 44, pp. 677-690, March 1998.

9. Y. Hua, M. Nikpour and P. Stoica, ”Optimal reduced rank estimation and filtering,” IEEE Transactions on Signal Processing, pp. 457-469, Vol. 49, No. 3, March 2001.

10. J. S. Goldstein and I. S. Reed, “Reduced rank adaptive filtering,” *IEEE Trans. Signal Processing*, vol. 45, pp. 492-496, Feb. 1997.

11. J. S. Goldstein, I. S. Reed and L. L. Scharf, “A multistage representation of the Wiener filter based on orthogonal projections,” *IEEE Transactions on Information Theory*, vol. 44, November, 1998.

12. M. L. Honig and J. S. Goldstein, “Adaptive reduced-rank interference suppression based on the multistage Wiener filter,” *IEEE Trans. on Communications*, vol. 50, no. 6, June, 2002.

13. R. C. de Lamare, M. Haardt and R. Sampaio-Neto, ”Blind Adaptive Constrained Reduced-Rank Parameter Estimation based on Constant Modulus Design for CDMA Interference Suppression,” IEEE Transactions on Signal Processing, vol. 56, no. 6, pp. 2470 - 2482, June 2008.

14. C. C. Hu and I. S. Reed, “Space-Time Adaptive Reduced-Rank Multistage Wiener Filtering for Asynchronous DS-CDMA,” *IEEE Trans. on Sig. Proc.*, vol. 52, no. 7, July, 2004.

15. S.-H. Wu ; U. Mitra and C.-C. J Kuo, “Reduced-rank multistage receivers for DS-CDMA in frequency-selective fading channels”, *IEEE Transactions on Communications*, vol. 53, no. 2, Feb. 2005.

16. R. C. de Lamare, M. Haardt and R. Sampaio-Neto, “Blind Adaptive Constrained Reduced-Rank Parameter Estimation based on Constant Modulus Design for CDMA Interference Suppression”, *IEEE Transactions on Signal Processing*, vol. 56, No. 6, June 2008.

17. W. Chen, U. Mitra, and P. Schniter, ” On the equivalence of three reduced rank linear estimators with applications to DS-CDMA,” *IEEE Transactions on Information Theory*, vol.48, pp. 2609-2614, September 2002.

18. M. Yukawa, R. C. de Lamare, and I. Yamada, “Robust Reduced-Rank Adaptive Algorithm Based on Parallel Subgradient Projection and Krylov Subspace”, *IEEE Transactions on Signal Processing*, vol. 57, no. 12, pp. 4660-4674, 2009.

N. Song, R. C. de Lamare, M. Haardt and M. Wolf, ”Adaptive Widely Linear Reduced-Rank Interference Suppression based on the Multi-Stage Wiener Filter”, IEEE Transactions on Signal Processing, 2012.

19. R. Singh and L. B. Milstein, “Adaptive interference suppression for DS/CDM
IEEE Transactions on Communications, vol. 50, no. 12, pp. 1902-1905, November, 2002.

20. D. A. Pados and S. N. Batalama, "Low complexity blind detection of DS/CDMA signals : Auxiliary-vector receivers," *IEEE Transactions on Communications*, vol. 45, pp. 1586-1594, December 1997.

21. D. A. Pados and S. N. Batalama, "Joint space-time auxiliary-vector filtering for DS/CDMA systems with antenna arrays," *IEEE Trans. on Communications*, vol. 47, no. 8, Sept. 1999.

22. D. A. Pados, F. J. Lombardo and S. N. Batalama, "Auxiliary Vector Filters and Adaptive Steering for DS-CDMA Single-User Detection," *IEEE Transactions on Vehicular Technology*, vol. 48, No. 6, November 1999.

23. D. A. Pados, G. N. Karystinos, "An iterative algorithm for the computation of the MVDR filter," *IEEE Trans. on Sig. Proc.*, vol. 49, No. 2, February, 2001.

24. H. Qian and S.N. Batalama, "Data record-based criteria for the selection of an auxiliary vector estimator of the MMSE/MVDR filter", *IEEE Trans. on Communications*, vol. 51, no. 10, Oct. 2003, pp. 1700 - 1708.

25. L. Wang, R. C. de Lamare, "Auxiliary Vector Filtering Algorithm Based on Constrained Constant Modulus Design for Robust Adaptive Beamforming", *IEEE Transactions on Signal Processing*, 2010.

26. R. C. de Lamare and R. Sampaio-Neto, "Adaptive Reduced-Rank MMSE Filtering with Interpolated FIR Filters and Adaptive Interpolators", *IEEE Signal Processing Letters*, vol. 12, no. 3, March, 2005.

27. R. C. de Lamare and R. Sampaio-Neto, “Adaptive Interference Suppression for DS-CDMA Systems based on Interpolated FIR Filters with Adaptive Interpolators in Multipath Channels”, *IEEE Trans. Vehicular Technology*, Vol. 56, no. 6, September 2007.

28. R. C. de Lamare and Raimundo Sampaio-Neto, “Reduced-Rank Adaptive Filtering Based on Joint Iterative Optimization of Adaptive Filters”, IEEE Signal Processing Letters, Vol. 14, no. 12, December 2007.

29. R. C. de Lamare and R. Sampaio-Neto, “Reduced-Rank Space-Time Adaptive Interference Suppression with Joint Iterative Least Squares Algorithms for Spread Spectrum Systems”, *IEEE Transactions on Vehicular Technology*, 2010.

30. R. Fa and R. C. de Lamare, ”Reduced-Rank STAP Using Joint Optimization Algorithms”, *IEEE Transactions on Aerospace and Electronic Systems*, 2010.

31. L. Wang, R. C. de Lamare and M.Yukawa, ”Adaptive Reduced-Rank Constrained Constant Modulus Algorithms Based on Joint Iterative Optimization of Filters for Beamforming”, *IEEE Transactions on Signal Processing*, vol. 58, no. 6, pp. 2983-2997, 2010.

32. P. Clarke and R. C. de Lamare, “Low-Complexity Reduced-Rank Linear Interference Suppression based on Set-Membership Joint Iterative Optimization for DS-CDMA Systems”, *IEEE Transactions on Vehicular Technology*, vol. 60, no. 9, 4324-4337, 2011

33. R. C. de Lamare and R. Sampaio-Neto, "Adaptive Space-Time Reduced-Rank Interference Suppression for Asynchronous DS-CDMA based on a Diversity Combined Decimation and Interpolation Scheme", *Proc. IEEE International Conference on Communications*, 2007.

34. R. C. de Lamare and R. Sampaio-Neto, "Adaptive Reduced-Rank MMSE Parameter Estimation based on an Adaptive Diversity Combined Decimation and Interpolation Scheme," *Proc. IEEE International Conference on Acoustics, Speech and Signal Processing*, 2007.

35. R. C. de Lamare and R. Sampaio-Neto, "Adaptive Reduced-Rank Processing Based on Joint and Iterative Interpolation, Decimation and Filtering", *IEEE Trans. on Signal Processing*, vol. 57, 2009.

36. R. Fa., R. C. de Lamare, and L. Wang, "Reduced-Rank STAP Schemes for Airborne Radar Based on Switched Joint Interpolation, Decimation and Filtering Algorithm", *IEEE Transactions on Signal Processing*, vol. 58, no. 8, pp. 4182-4194, 2010.

37. R. C. de Lamare, R. Sampaio-Neto, M. Haardt, "Blind Adaptive Reduced-Rank Estimation Based on The Constant Modulus Criterion and Diversity-Combined Decimation and Interpolation", *Proc. IEEE ICASSP*, 2008.

38. R. C. de Lamare, R. Sampaio-Neto and M. Haardt, "Blind Adaptive Constrained Constant-Modulus Reduced-Rank Interference Suppression Algorithms Based on Interpolation and Switched Decimation", *IEEE Transactions on Signal Processing*, vol. 59, no. 2, pp. 681-695, 2011.

39. S. Li and R. C. de Lamare, "Reduced-Rank Linear Interference Suppression for DS-UWB Systems Based on Switched Approximations of Adaptive Basis Functions", IEEE Transactions on Vehicular Technology, vol. 60, no. 2, pp. 485-497, 2011.

40. P Stoica, Y Selen, " Model-order selection : a review of information criterion rules", IEEE Signal Processing Magazine, vol. 21, no. 4, pp. 36-47, 2004.

41. H. Akaike, "A new look at statistical model identification,"IEEE Trans. Automat. Contr., vol. 19, pp. 716-723, Dec. 1974.

42. J. Rissanen, "Estimation of structure by minimum description length," Circuits, Syst. Signal Process., vol. 1, no. 4, pp. 395-406, 1982.