Blind Reduced-Rank Adaptive Receivers for DS-UWB Systems Based on Joint Iterative Optimization and the Constrained Constant Modulus Criterion

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***Abstract*—A novel linear blind adaptive receiver based on joint iterative optimization (JIO) and the constrained-constant-modulus design criterion is proposed for interference suppression in direct-sequence ultrawideband systems. The proposed blind receiver consists of the following two parts: 1) a transformation matrix that performs dimensionality reduction and 2) a reduced-rank filter that produces the output. In the proposed receiver, the transformation matrix and the reduced-rank filter are jointly and iteratively updated to minimize the constant-modulus cost function subject to a constraint. Adaptive implementations for the JIO receiver are developed using the normalized stochastic gradient (NSG) and recursive least squares (RLS) algorithms. To obtain a low-complexity scheme, the columns of the transformation matrix with the RLS algorithm are individually updated. Blind channel estimation algorithms for both versions (i.e., NSG and RLS) are implemented. Assuming perfect timing, the JIO receiver only requires the spreading code of the desired user and the received data. Simulation results show that both versions of the proposed JIO receivers have excellent performance in terms of suppressing the intersymbol interference (ISI) and multiple-access interference (MAI) with low complexity.**

***Index Terms*—Blind adaptive receiver, constrained constant modulus (CCM), direct-sequence ultrawideband (DS-UWB) systems, interference suppression, reduced-rank methods.**

I. INTRODUCTION

ULTRAWIDEBAND (UWB) technology [1]–[5], which can achieve very high data rates, is a promising short-range wireless communication technique. By spreading the information symbols with a pseudorandom (PR) code, the direct sequence ultrawideband (DS-UWB) technique enables multiuser communications [6]. In DS-UWB systems, a high degree of diversity is achieved at the receiver due to the large number of resolvable multipath components (MPCs) [7]. Receivers are required to efficiently suppress the severe intersymbol interference (ISI), which is caused by the dense multipath channel, and the multiple-access interference (MAI), which is caused by the lack of orthogonality between signals at the receiver in multiuser communications.

Blind adaptive linear receivers [8]–[14] are efficient schemes for interference suppression, because they offer higher spectrum efficiency than the adaptive schemes, which require a training stage. Low-complexity blind receiver designs can be obtained by solving constrained optimization problems based on the constrained constant modulus (CCM) or constrained minimum variance (CMV) criterion [12], [15]. Blind receiver designs based on the CCM criterion have shown better performance and increased robustness to signature mismatches over CMV approaches [12], [14]. Recently, blind full-rank stochastic gradient (SG) and recursive least squares (RLS) adaptive filters based on the constrained optimization have been proposed for multiuser detection in DS-UWB communications [15], [16]. For DS-UWB systems in which the received signal length is large due to the long channel delay spread, the interference-sensitive full-rank adaptive schemes experience a slow convergence rate. In large-filter scenarios, reduced-rank algorithms can be adopted to accelerate the convergence and provide an increased robustness to interference and noise.

By projecting the received signal onto a lower dimensional subspace and adapting a lower order filter to process the reduced-rank signal, the reduced-rank filters can achieve an increased robustness and faster convergence than full-rank schemes [17]–[32]. The existing reduced-rank schemes include the eigendecomposition methods and the Krylov subspace schemes. The eigendecomposition methods include the principal components (PC) [17] and the cross-spectral metric (CSM) [18] approaches, which are based on the eigendecomposition of the estimated covariance matrix of the received signal. In the PC scheme, the received signal is projected onto a subspace that is associated with the largest eigenvalues [19], and in the CSM approach, the subspace is selected with the maximum signal-to-interference-plus-noise ratio (SINR) [20]. It is known that the optimal representation of the input data can be obtained by the eigendecomposition of its covariance matrix $\mathbf{R}$ [21]. However, these methods have very high computational complexity, and the robustness to interference is often poor in heavily loaded communication systems [19]. The Krylov subspace schemes

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include the powers of R (POR) [22], the multistage Wiener filter (MSWF) [19], [21], and the auxiliary vector filtering (AVF) [23]. All these schemes project the received signal onto the Krylov subspace [22] and achieve faster convergence speed than the full-rank schemes with a smaller filter size. However, the high computational complexity is also a problem of the Krylov subspace methods.

For UWB systems, reduced-rank receivers that require training sequences have recently been developed in [33]–[37]. Solutions for reduced-rank channel estimation and synchronization in single-user UWB systems have been proposed in [33]. For multiuser detection in UWB communications, reduced-rank schemes that require the knowledge of the multipath channel have been developed in [34]–[36]. We proposed a low-complexity reduced-rank interference suppression scheme for DS-UWB systems that can efficiently suppress both the ISI and MAI in [37]. In [38], a blind subspace multiuser detection scheme is proposed for UWB systems, which requires the eigendecomposition of the covariance matrix of the received signal. In this paper, a novel CCM-based joint iterative optimization (JIO) blind reduced-rank receiver is proposed. A transformation matrix and a reduced-rank filter construct the proposed receiver, and they are jointly and iteratively updated to minimize the CM cost function subject to a constraint. The proposed receiver allows information exchange between the transformation matrix and the reduced-rank filter. This distinguishing feature leads to a more efficient adaptive implementation than the existing reduced-rank schemes. Note that the constraint is necessary, because it enables us to avoid the undesired local minima. The adaptive normalized stochastic gradient (NSG) and RLS algorithms are developed for the JIO receiver. In the NSG version, a low-complexity leakage SG channel estimator that was proposed in [43] is adopted. Applying an approximation to the covariance matrix of the received signal, the RLS channel estimator that was proposed in [43] is modified for the proposed JIO–RLS with reduced complexity. Because each column of the transformation matrix can be considered a direction vector on one dimension of the subspace, we update the transformation matrix column by column to achieve a better representation of the projection procedure in the JIO–RLS.

The main contributions of this paper are summarized as follows.

- A novel linear blind JIO reduced-rank receiver based on the CCM criterion is proposed for interference suppression in DS-UWB systems.
- NSG algorithms, which can facilitate the setting of step sizes in multiuser scenarios, are developed for the proposed reduced-rank receivers.
- RLS algorithms are developed to jointly update the columns of the transformation matrix and the reduced-rank filter with low complexity.
- A rank adaptation algorithm is developed to achieve a better tradeoff between the convergence speed and the steady-state performance.
- The convergence properties of the CM cost function with a constraint are discussed.
- Simulations are performed with the IEEE 802.15.4a channel models, and severe ISI and MAI are assumed for the evaluation of the proposed scheme against existing techniques.

The rest of this paper is structured as follows. Section II presents the DS-UWB system model. The design of the JIO CCM blind receiver is detailed in Section III. The proposed NSG and RLS versions of the blind JIO receiver are described in Sections IV and V, respectively. In Section VI, a complexity analysis for the proposed receiver versions is detailed, and a rank adaptation algorithm is developed for the JIO receiver. Simulation results are shown in Section VII, and conclusions are drawn in Section VIII.

II. Direct Sequence-Ultrawideband System Model

In this paper, we consider the uplink of a binary phase-shift keying (BPSK) DS-UWB system with K users. A random spreading code $\mathbf{s}_k$ is assigned to the kth user with a spreading gain $N_c = T_s/T_c$, where T_s and T_c denote the symbol duration and chip duration, respectively. The transmit signal of the kth user (where $k = 1, 2, \ldots, K$) can be expressed as

$$x^{(k)}(t) = \sqrt{E_k} \sum_{i=-\infty}^{\infty} \sum_{j=0}^{N_c-1} p_t(t - iT_s - jT_c) s_k(j) b_k(i) \quad (1)$$

where $b_k(i) \in \{\pm 1\}$ denotes the BPSK symbol for the kth user at the ith time instant, and $s_k(j)$ denotes the jth chip of the spreading code $\mathbf{s}_k$ (where $j = 1, 2, \ldots, N_c$). E_k denotes the transmission energy of the kth user. $p_t(t)$ is the pulse waveform of width T_c. Throughout this paper, the pulse waveform $p_t(t)$ is modeled as the root-raised cosine (RRC) pulse with a roll-off factor of 0.5 [39], [40]. The channel model that we consider is the IEEE 802.15.4a channel model for the indoor residential environment [41]. This standard channel model includes some generalizations of the Saleh–Valenzuela model and takes the frequency dependence of the path gain into account [42]. In addition, the IEEE 802.15.4a channel model is valid for both low- and high-data-rate UWB systems [42]. For the kth users, the channel impulse response (CIR) of the standard channel model can be expressed as

$$h_k(t) = \sum_{u=0}^{L_c-1} \sum_{v=0}^{L_r-1} \alpha_{u,v} e^{j\phi_{u,v}} \delta(t - T_u - T_{u,v}) \quad (2)$$

where L_c denotes the number of clusters, and L_r is the number of MPCs in one cluster. $\alpha_{u,v}$ is the fading gain of the vth MPC in the uth cluster, and $\phi_{u,v}$ is uniformly distributed in $[0, 2\pi)$. T_u is the arrival time of the uth cluster, and $T_{u,v}$ denotes the arrival time of the vth MPC in the uth cluster. For simplicity, we express the CIR as

$$h_k(t) = \sum_{l=0}^{L-1} h_{k,l} \delta(t - lT_\tau) \quad (3)$$

where $h_{k,l}$ and lT_τ present the complex-valued fading factor and the arrival time of the lth MPC ($l = uL_c + v$), respectively.

$L = T_{DS}/T_\tau$ denotes the total number of MPCs, where T_{DS} is the channel delay spread. Assuming that the timing is acquired, the received signal can be expressed as

$$z(t) = \sum_{k=1}^{K}\sum_{l=0}^{L-1} h_{k,l}x^{(k)}(t - lT_\tau) + n(t) \quad (4)$$

where $n(t)$ is the additive white Gaussian noise (AWGN) with zero mean and a variance of σ_n^2. This signal is first passed through a chip matched filter (MF) and then sampled at the chip rate. We select a total number of $M = (T_s + T_{DS})/T_c$ observation samples for the detection of each data bit, where T_s is the symbol duration, T_{DS} is the channel delay spread, and T_c is the chip duration. Assuming that the sampling starts at the zeroth time instant, the mth sample is given by

$$r_m = \int_{mT_c}^{(m+1)T_c} z(t)p_r(t)dt, \qquad m = 1, 2, \ldots, M$$

where $p_r(t) = p_t^*(-t)$ denotes the chip MF, and $(\cdot)^*$ denotes the complex conjugation. After the chip-rate sampling, the discrete-time received signal for the ith data bit can be expressed as $\mathbf{r}(i) = [r_1(i), r_2(i), \ldots, r_M(i)]^T$, where $(\cdot)^T$ is the transposition, and we can further express it in matrix form as

$$\mathbf{r}(i) = \sum_{k=1}^{K} \sqrt{E_k}\mathbf{P}_r\mathbf{H}_k\mathbf{P}_t\mathbf{s}_k b_k(i) + \boldsymbol{\eta}(i) + \mathbf{n}(i) \quad (5)$$

where $\mathbf{H}_k$ is the Toeplitz channel matrix for the kth user, with the first column being the CIR $\mathbf{h}_k = [h_k(0), h_k(1), \ldots, h_k(L-1)]^T$ that is zero padded to length $M_H = (T_s/T_\tau) + L - 1$. The matrix $\mathbf{P}_r$ represents the MF and chip-rate sampling with a size of $M \times M_H$. $\mathbf{P}_t$ denotes the $(T_s/T_\tau) \times N_c$ pulse-shaping matrix. To facilitate the blind channel estimation (BCE) in a later development, we rearrange the term and express the received signal as

$$\mathbf{r}(i) = \sum_{k=1}^{K} \sqrt{E_k}\mathbf{P}_r\mathbf{S}_{e,k}\mathbf{h}_k b_k(i) + \boldsymbol{\eta}(i) + \mathbf{n}(i) \quad (6)$$

where $\mathbf{S}_{e,k}$ is the Toeplitz matrix, with the first column being the vector $\mathbf{s}_{e,k} = \mathbf{P}_t\mathbf{s}_k$ that is zero padded to length M_H. The vector $\boldsymbol{\eta}(i)$ denotes the ISI from $2G$ adjacent symbols, where G denotes the minimum integer that is larger than or equal to the scalar term T_{DS}/T_s. Here, we express the ISI vector in the general form as

$$\boldsymbol{\eta}(i) = \sum_{k=1}^{K}\sum_{g=1}^{G} \sqrt{E_k}\mathbf{P}_r\mathbf{H}_k^{(-g)}\mathbf{P}_t\mathbf{s}_k b_k(i-g) + \sum_{k=1}^{K}\sum_{g=1}^{G} \sqrt{E_k}\mathbf{P}_r\mathbf{H}_k^{(+g)}\mathbf{P}_t\mathbf{s}_k b_k(i+g) \quad (7)$$

where the channel matrices for the ISI are given by

$$\mathbf{H}_k^{(-g)} = \begin{bmatrix} \mathbf{0} & \mathbf{H}_k^{(u,g)} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}; \quad \mathbf{H}_k^{(+g)} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{H}_k^{(l,g)} & \mathbf{0} \end{bmatrix}. \quad (8)$$

Note that the matrices $\mathbf{H}_k^{(u,g)}$ and $\mathbf{H}_k^{(l,g)}$ have the same size as $\mathbf{H}_k$, which is $M_H \times (T_s/T_\tau)$ and can be considered the partitions of an upper $\mathbf{H}_{\text{up}}$ and a lower triangular matrix $\mathbf{H}_{\text{low}}$, respectively, where

$$\mathbf{H}_{\text{up}} = \begin{bmatrix} h_k(L-1) & \cdots & h_k\left(L - \frac{T_{DS}-(g-1)T_s}{T_\tau}\right) \\ & \ddots & \vdots \\ & & h_k(L-1) \end{bmatrix}$$

$$\mathbf{H}_{\text{low}} = \begin{bmatrix} h_k(0) & & \\ \vdots & \ddots & \\ h_k\left(\frac{T_{DS}-(g-1)T_s}{T_\tau} - 2\right) & \cdots & h_k(0) \end{bmatrix}.$$

These triangular matrices have the row dimension of $[T_{DS} - (g-1)T_s]/T_\tau - 1 = L - (g-1)T_s/T_\tau - 1$. Note that, when the channel delay spread is large, the row dimension of these triangular matrices can surpass the column dimension of the matrix $\mathbf{H}_k$, which is T_s/T_τ. Hence, in the case of

$$L - (g-1)T_s/T_\tau - 1 > T_s/T_\tau$$
$$\text{i.e.,} \quad L > gT_s/T_\tau + 1 \quad (9)$$

the matrix $\mathbf{H}_k^{(u,g)}$ is the last T_s/T_τ columns of the upper triangular matrix $\mathbf{H}_{\text{up}}$, and $\mathbf{H}_k^{(l,g)}$ is the first T_s/T_τ columns of the lower triangular matrix $\mathbf{H}_{\text{low}}$. When $L < gT_s/T_\tau + 1$, $\mathbf{H}_k^{(u,g)} = \mathbf{H}_{\text{up}}$, and $\mathbf{H}_k^{(l,g)} = \mathbf{H}_{\text{low}}$. It is interesting to review the expression of the ISI vector through its physical meaning, because the row dimension of the matrices $\mathbf{H}_k^{(u,g)}$ and $\mathbf{H}_k^{(l,g)}$, which is $L - (g-1)T_s/T_\tau - 1$, reflects the time-domain overlap between the data symbol $b(i)$ and the adjacent symbols of $b(i-g)$ and $b(i+g)$.

III. Proposed Blind Joint Iterative Optimization Reduced-Rank Receiver Design

In this section, we detail the design of the proposed JIO reduced-rank receiver, which can blindly recover the data symbol from the noisy received signal. The block diagram of the proposed receiver is shown in Fig. 1. In the JIO blind linear receiver, the reduced-rank received signal can be expressed as

$$\bar{\mathbf{r}}(i) = \mathbf{T}^H(i)\mathbf{r}(i) \quad (10)$$

where $\mathbf{T}(i)$ is the $M \times D$ (where $D \ll M$) transformation matrix. After the projection, $\bar{\mathbf{r}}(i)$ is fed into the reduced-rank filter $\bar{\mathbf{w}}(i)$, and the output signal is given by

$$y(i) = \bar{\mathbf{w}}^H(i)\bar{\mathbf{r}}(i). \quad (11)$$

The decision of the desired data symbol is defined as

$$\hat{b}(i) = \text{sign}\left(\Re\left[y(i)\right]\right) \quad (12)$$

where $\text{sign}(\cdot)$ is the algebraic sign function, and $\Re(\cdot)$ represents the real part of a complex number.

The optimization problem to be solved can be expressed as

$$[\bar{\mathbf{w}}(i), \mathbf{T}(i)] = \arg\min_{\bar{\mathbf{w}}(i),\mathbf{T}(i)} \mathbf{J}_{\text{JIO}}\left(\bar{\mathbf{w}}(i), \mathbf{T}(i)\right) \quad (13)$$

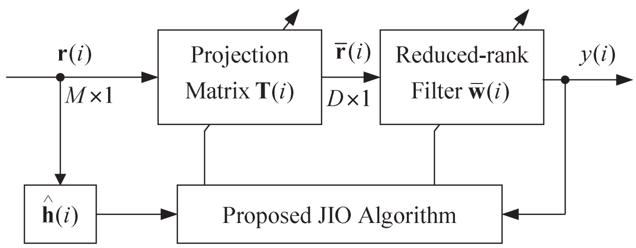

Fig. 1. Block diagram of the proposed blind reduced-rank receiver.

subject to the constraint

$$\bar{\mathbf{w}}^H(i)\mathbf{T}^H(i)\mathbf{p} = v \tag{14}$$

where $\mathbf{p} = \mathbf{P}_r\mathbf{S}_e\mathbf{h}$ is defined as the effective signature vector for the desired user, and v is a real-valued constant to ensure the convexity of the CM cost function

$$\mathbf{J}_{\text{JIO}}\left(\bar{\mathbf{w}}(i), \mathbf{T}(i)\right) = \frac{1}{2}E\left[\left(|y(i)|^2 - 1\right)^2\right]. \tag{15}$$

The convergence properties of the CM cost function subject to a constraint are discussed in Appendix.

Let us now consider the problem through the Lagrangian

$$\mathcal{L}_{\text{JIO}}\left(\bar{\mathbf{w}}(i), \mathbf{T}(i)\right) = \frac{1}{2}E\left[\left(|y(i)|^2 - 1\right)^2\right] + \Re\left[\lambda(i)\left(\bar{\mathbf{w}}^H(i)\mathbf{T}^H(i)\mathbf{p} - v\right)\right] \tag{16}$$

where $\lambda(i)$ is a complex-valued Lagrange multiplier. To obtain the adaptation equation of $\mathbf{T}(i)$, we first assume that $\bar{\mathbf{w}}(i)$ is fixed and the gradient of the Lagrangian with respect to $\mathbf{T}(i)$ is given by

$$\nabla_T\mathcal{L}_{\text{JIO}} = E\left[e(i)y^*(i)\mathbf{r}(i)\bar{\mathbf{w}}^H(i)\right] + \frac{\lambda_T(i)}{2}\mathbf{p}\bar{\mathbf{w}}^H(i) \tag{17}$$

where $\lambda_T(i)$ is the complex-valued Lagrange multiplier for updating the transformation matrix, and $e(i) = |y(i)|^2 - 1$ is defined as a real-valued error signal. Recalling the relationship $y^*(i) = \mathbf{r}^H(i)\mathbf{T}(i)\bar{\mathbf{w}}(i)$ and setting (17) to a zero matrix, we obtain

$$\mathbf{T}_{\text{opt}} = \mathbf{R}_Y^{-1}\left(\mathbf{D}_T - \frac{\lambda_T(i)}{2}\mathbf{p}\bar{\mathbf{w}}^H(i)\right)\mathbf{R}_w^{-1} \tag{18}$$

where $\mathbf{R}_Y = E[|y(i)|^2\mathbf{r}(i)\mathbf{r}^H(i)]$, $\mathbf{D}_T = E[y^*(i)\mathbf{r}(i)\bar{\mathbf{w}}^H(i)]$, and $\mathbf{R}_w = E[\bar{\mathbf{w}}(i)\bar{\mathbf{w}}^H(i)]$. Using the constraint $\bar{\mathbf{w}}^H(i)\mathbf{T}_{\text{opt}}^H\mathbf{p} = v$, we obtain the Lagrange multiplier

$$\lambda_T(i) = 2\left(\frac{\bar{\mathbf{w}}^H(i)\mathbf{R}_w^{-1}\mathbf{D}_T\mathbf{R}_Y^{-1}\mathbf{p} - v}{\bar{\mathbf{w}}^H(i)\mathbf{R}_w^{-1}\bar{\mathbf{w}}(i)\mathbf{p}^H\mathbf{R}_Y^{-1}\mathbf{p}}\right)^*. \tag{19}$$

Now, we assume that $\mathbf{T}(i)$ is fixed in (16) and calculate the gradient of the Lagrangian with respect to $\bar{\mathbf{w}}(i)$, which is given by

$$\nabla_w\mathcal{L}_{\text{JIO}} = E\left[e(i)\mathbf{T}^H(i)\mathbf{r}(i)y^*(i)\right] + \frac{\lambda_w(i)}{2}\mathbf{T}^H(i)\mathbf{p} \tag{20}$$

where $\lambda_w(i)$ is the complex-valued Lagrange multiplier for updating the reduced-rank filter. Rearranging the terms, we obtain

$$\bar{\mathbf{w}}_{\text{opt}} = \mathbf{R}_{\bar{y}}^{-1}\left(\mathbf{d}_{\bar{r}} - \frac{\lambda_w(i)}{2}\mathbf{T}^H(i)\mathbf{p}\right) \tag{21}$$

where $\mathbf{R}_{\bar{y}} = E[|y(i)|^2\bar{\mathbf{r}}(i)\bar{\mathbf{r}}^H(i)]$, and $\mathbf{d}_{\bar{r}} = E[y^*(i)\bar{\mathbf{r}}(i)]$. Using the constraint $\bar{\mathbf{w}}_{\text{opt}}^H\mathbf{T}^H(i)\mathbf{p} = v$, we obtain the Lagrange multiplier

$$\lambda_w(i) = 2\left(\frac{\mathbf{d}_{\bar{r}}^H\mathbf{R}_{\bar{y}}^{-1}\mathbf{T}^H(i)\mathbf{p} - v}{\mathbf{p}^H\mathbf{T}(i)\mathbf{R}_{\bar{y}}^{-1}\mathbf{T}^H(i)\mathbf{p}}\right)^*. \tag{22}$$

With the solutions of $\mathbf{T}_{\text{opt}}$ and $\bar{\mathbf{w}}_{\text{opt}}$, the NSG and RLS adaptive versions of the JIO receiver will be developed in the following sections, in which the direct matrix inversions are not required, and the computational complexity is reduced. Note that, when adaptive algorithms are implemented to estimate $\mathbf{T}_{\text{opt}}$ and $\bar{\mathbf{w}}_{\text{opt}}$, $\mathbf{T}(i)$ is a function of $\bar{\mathbf{w}}(i)$ and $\bar{\mathbf{w}}(i)$ is a function of $\mathbf{T}(i)$. Thus, the optimal CCM design is not in a closed form, and one possible solution for such an optimization problem is to jointly and iteratively adapt these two quantities. The joint update means for the ith time instant $\mathbf{T}(i)$ is updated with the knowledge of $\mathbf{T}(i-1)$ and $\bar{\mathbf{w}}(i-1)$, and then, $\bar{\mathbf{w}}(i)$ is updated with $\mathbf{T}(i)$ and $\bar{\mathbf{w}}(i-1)$. Each iterative update can be considered one repetition of the joint update.

Note that the blind JIO receiver design requires the knowledge of the effective signature vector of the desired user or, equivalently, the channel parameters. In this paper, the channel coefficients are not given and must be estimated. Here, we employ the variant of the power method introduced in [43], i.e.,

$$\hat{\mathbf{h}}(i) = \left(\mathbf{I} - \hat{\mathbf{V}}(i)/tr[\hat{\mathbf{V}}(i)]\right)\hat{\mathbf{h}}(i-1) \tag{23}$$

where the $L \times L$ matrix is defined as

$$\hat{\mathbf{V}}(i) = \mathbf{S}_e^H\mathbf{P}_r^H\mathbf{R}^{-m}(i)\mathbf{P}_r\mathbf{S}_e \tag{24}$$

$\mathbf{I}$ is the identity matrix, $tr[\cdot]$ stands for the trace, and we make $\hat{\mathbf{h}}(i) \leftarrow \hat{\mathbf{h}}(i)/\|\hat{\mathbf{h}}(i)\|$ to normalize the channel. $\mathbf{R}(i) = \sum_{j=1}^{i}\alpha^{i-j}\mathbf{r}(j)\mathbf{r}^H(j)$, and m is a finite power. The estimate of the matrix $\mathbf{R}^{-1}(i)$ is recursively obtained through the matrix inversion lemma [44] and is given by

$$\hat{\mathbf{R}}^{-1}(i) = \frac{1}{\alpha}\left(\hat{\mathbf{R}}^{-1}(i-1) - (\phi(i)\boldsymbol{\kappa}(i))\,\boldsymbol{\kappa}^H(i)\right) \tag{25}$$

where α is the forgetting factor, $\boldsymbol{\kappa}(i) = \hat{\mathbf{R}}^{-1}(i-1)\mathbf{r}(i)$, and $\phi(i) = (\alpha + \mathbf{r}^H(i)\boldsymbol{\kappa}(i))^{-1}$. The estimation of the inversion of the covariance matrix requires $3M^2 + 2M + 1$ multiplications and $2M^2$ additions. Equation (24) requires $(m+1)M^2L$ multiplications and $(m+1)M^2L - (m+1)ML$ additions, whereas (23) requires L^2 multiplications and $L^2 + L - 1$ additions (the multiplications and additions in this paper are both complex-valued operations). Note that the matrix $\mathbf{P}_r\mathbf{S}_e$ is assumed given at the receiver.

The estimate of the effective signature vector can finally be obtained as

$$\hat{\mathbf{p}}(i) = \mathbf{P}_r\mathbf{S}_e\hat{\mathbf{h}}(i) \tag{26}$$

where $\hat{\mathbf{h}}(i)$ is given in (23).

IV. Proposed Joint Iterative Optimization–Normalized Stochastic Gradient Algorithms

In this section, we develop the NSG algorithm to jointly and iteratively update $\mathbf{T}(i)$ and $\bar{\mathbf{w}}(i)$. The BCE based on the leakage SG algorithm that is proposed in [43] is implemented to provide the channel coefficients.

A. JIO–NSG Algorithms

The optimization problem to be solved in the NSG version is given by

$$[\bar{\mathbf{w}}(i), \mathbf{T}(i)] = \arg \min_{\bar{\mathbf{w}}(i), \mathbf{T}(i)} \mathbf{J}_{\mathrm{JIO}} \left(\bar{\mathbf{w}}(i), \mathbf{T}(i)\right) \tag{27}$$

subject to $\bar{\mathbf{w}}^H(i)\mathbf{T}^H(i)\hat{\mathbf{p}}(i) = v$, where $\hat{\mathbf{p}}(i)$ is the estimated signature vector that is obtained through BCE, which will be detailed in Section IV-B, and v is a real-valued constant to ensure the convexity of the cost function

$$\mathbf{J}_{\mathrm{JIO-NSG}} \left(\bar{\mathbf{w}}(i), \mathbf{T}(i)\right) = \frac{1}{2} E\left[\left(|y(i)|^2 - 1\right)^2\right]. \tag{28}$$

Here, we consider the problem through the Lagrangian

$$\mathcal{L}_{\mathrm{JIO-NSG}} \left(\bar{\mathbf{w}}(i), \mathbf{T}(i)\right) = \frac{1}{2} E\left[\left(|y(i)|^2 - 1\right)^2\right] + \Re\left[\lambda_N(i)\left(\bar{\mathbf{w}}^H(i)\mathbf{T}^H(i)\hat{\mathbf{p}}(i) - v\right)\right] \tag{29}$$

where $\lambda_N(i)$ is a complex-valued Lagrange multiplier. For each time instant, we first update $\mathbf{T}(i)$ while assuming that $\bar{\mathbf{w}}(i)$ is fixed. Then, we adapt $\bar{\mathbf{w}}(i)$ with the updated $\mathbf{T}(i)$.

The gradient of the Lagrangian with respect to $\mathbf{T}(i)$ is given by

$$\nabla_T \mathcal{L}_{\mathrm{JIO-NSG}} = E\left[e(i)y^*(i)\mathbf{r}(i)\bar{\mathbf{w}}^H(i)\right] + \frac{1}{2}\lambda_{NT}(i)\hat{\mathbf{p}}(i)\bar{\mathbf{w}}^H(i)$$

where $\lambda_{NT}(i)$ is the complex-valued Lagrange multiplier for updating the transformation matrix, and $e(i) = |y(i)|^2 - 1$ is defined as a real-valued error signal. Using the instantaneous estimator to the gradient vector, the SG update equation is given by

$$\mathbf{T}(i+1) = \mathbf{T}(i) - \mu_T\left(e(i)y^*(i)\mathbf{r}(i) + \frac{\lambda_{NT}(i)}{2}\hat{\mathbf{p}}(i)\right)\bar{\mathbf{w}}^H(i) \tag{30}$$

where μ_T is the step size for the SG algorithm that updates the transformation matrix. Using the constraint of $\bar{\mathbf{w}}^H(i)\mathbf{T}^H(i+1)\hat{\mathbf{p}}(i) = v$, we obtain

$$\lambda_{NT}(i) = 2\frac{\hat{\mathbf{p}}^H(i)\mathbf{T}(i)\bar{\mathbf{w}}(i) - \mu_T e(i)y^*(i)\left\|\bar{\mathbf{w}}(i)\right\|^2\hat{\mathbf{p}}^H(i)\mathbf{r}(i) - v}{\mu_T\left\|\bar{\mathbf{w}}(i)\right\|^2\left\|\hat{\mathbf{p}}(i)\right\|^2}. \tag{31}$$

The NSG algorithm aims at minimizing the cost function

$$\mathbf{J}_{\mathrm{JIO-NSG}}(\mu_T) = \frac{1}{2}\left[\left|\bar{\mathbf{w}}^H(i)\mathbf{T}^H(i+1)\mathbf{r}(i)\right|^2 - 1\right]^2. \tag{32}$$

Substituting (30) and (31) into (32) and setting the gradient vector of (32) with respect to μ_T to zeros, we obtain the solutions

$$\mu_{T,1} = \frac{|y(i)| - 1}{|y(i)|\, e(i)A_{T,1}}, \quad \mu_{T,2} = \frac{|y(i)| + 1}{|y(i)|\, e(i)A_{T,1}}$$
$$\mu_{T,3} = \mu_{T,4} = \frac{1}{e(i)A_{T,1}}$$

where the real-valued scale term $A_{T,1}$ is defined as

$$A_{T,1} = \left\|\bar{\mathbf{w}}(i)\right\|^2\left[\left\|\mathbf{r}(i)\right\|^2 - \frac{\left|\mathbf{r}^H(i)\hat{\mathbf{p}}(i)\right|^2}{\left\|\hat{\mathbf{p}}(i)\right\|^2}\right].$$

By examining the second derivative of (32) with respect to μ_T, we conclude that $\mu_{T,1}$ and $\mu_{T,2}$ are the solutions that correspond to the minima. In this paper, $\mu_{T,1}$ is used, and a positive real scaling factor $\mu_{T,0}$ is implemented, which will not change the direction of the tap-weight vector. Finally, the NSG update function of $\mathbf{T}(i)$ is given by

$$\mathbf{T}(i+1) = \mathbf{T}(i) - y^*(i)\mu_{T,0}\mathbf{A}_{T,2} - A_{T,3}\hat{\mathbf{p}}(i)\bar{\mathbf{w}}^H(i) \tag{33}$$

where

$$\mathbf{A}_{T,2} = \frac{|y(i)| - 1}{|y(i)|\, A_{T,1}}\left(\mathbf{r}(i)\bar{\mathbf{w}}^H(i) - \frac{\hat{\mathbf{p}}^H(i)\mathbf{r}(i)}{\left\|\hat{\mathbf{p}}(i)\right\|^2}\hat{\mathbf{p}}(i)\bar{\mathbf{w}}^H(i)\right)$$
$$A_{T,3} = \left(\left\|\bar{\mathbf{w}}(i)\right\|^2\left\|\hat{\mathbf{p}}(i)\right\|^2\right)^{-1}\left(\hat{\mathbf{p}}^H(i)\mathbf{T}(i)\bar{\mathbf{w}}(i) - v\right).$$

Now, let us adapt $\bar{\mathbf{w}}(i)$ while assuming that $\mathbf{T}(i)$ is fixed. The gradient of the Lagrangian with respect to $\bar{\mathbf{w}}(i)$ is given by $\nabla_w \mathcal{L}_{\mathrm{JIO-NSG}} = E[e(i)y^*(i)\mathbf{T}^H(i)\mathbf{r}(i)] + (1/2)\lambda_{Nw}(i)\hat{\mathbf{p}}(i)\bar{\mathbf{w}}^H(i)$, where $\lambda_{Nw}(i)$ is the complex-valued Lagrange multiplier for updating the reduced-rank filter. By using the instantaneous estimator of the gradient vector, the SG adaptation equation is given by

$$\bar{\mathbf{w}}(i+1) = \bar{\mathbf{w}}(i) - \mu_w e(i)y^*(i)\mathbf{T}^H(i)\mathbf{r}(i) - \mu_w\frac{\lambda_{Nw}(i)}{2}\mathbf{T}^H(i)\hat{\mathbf{p}}(i). \tag{34}$$

Using the constraint $\bar{\mathbf{w}}^H(i+1)\mathbf{T}^H(i)\hat{\mathbf{p}}(i) = v$, we have

$$\lambda_{Nw}(i) = 2\frac{\hat{\mathbf{p}}^H(i)\mathbf{T}(i)\bar{\mathbf{w}}(i) - \mu_w e(i)y^*(i)\hat{\mathbf{p}}^H(i)\mathbf{T}(i)\mathbf{T}^H(i)\mathbf{r}(i) - v}{\mu_w\left\|\mathbf{T}^H(i)\hat{\mathbf{p}}(i)\right\|^2}. \tag{35}$$

The NSG algorithm for updating the reduced-rank filter aims at minimizing the cost function

$$\mathbf{J}_{\mathrm{JIO-NSG}}(\mu_w) = \frac{1}{2}\left[\left|\bar{\mathbf{w}}^H(i+1)\mathbf{T}^H(i)\mathbf{r}(i)\right|^2 - 1\right]^2. \tag{36}$$

Substituting (34) and (35) into (36), the solutions of μ_w that correspond to a null gradient vector of (36) are given by

$$\mu_{w,1} = \frac{|y(i)| - 1}{|y(i)|\, e(i)A_{w,1}}, \quad \mu_{w,2} = \frac{|y(i)| + 1}{|y(i)|\, e(i)A_{w,1}}$$
$$\mu_{w,3} = \mu_{w,4} = \frac{1}{e(i)A_{w,1}}$$

where the scale term is given by

$$A_{w,1} = \left\|\mathbf{T}^H(i)\mathbf{r}(i)\right\|^2 - \frac{\left|\mathbf{r}^H(i)\mathbf{T}(i)\mathbf{T}^H(i)\hat{\mathbf{p}}(i)\right|^2}{\|\mathbf{T}^H(i)\hat{\mathbf{p}}(i)\|^2}.$$

By examining the second derivative of (36) with respect to μ_w, only $\mu_{w,1}$ and $\mu_{w,2}$ correspond to the minima of the cost function in (36). Finally, by applying a positive real scaling factor $\mu_{w,0}$ to control the tap-weight vector, the adaptation equation by using $\mu_{w,1}$ is given by

$$\bar{\mathbf{w}}^H(i+1) = \bar{\mathbf{w}}^H(i) - y^*(i)\mu_{w,0}\mathbf{A}_{w,2} - A_{w,3}\mathbf{T}^H(i)\hat{\mathbf{p}}(i) \quad (37)$$

where

$$\mathbf{A}_{w,2} = \frac{|y(i)|-1}{|y(i)|\,A_{w,1}} \times \left(\mathbf{T}^H(i)\mathbf{r}(i) - \frac{\hat{\mathbf{p}}^H(i)\mathbf{T}(i)\mathbf{T}^H(i)\mathbf{r}(i)}{\|\mathbf{T}^H(i)\hat{\mathbf{p}}(i)\|^2}\mathbf{T}^H(i)\hat{\mathbf{p}}(i)\right)$$

$$A_{w,3} = \left(\left\|\mathbf{T}^H(i)\hat{\mathbf{p}}(i)\right\|^2\right)^{-1}\left(\hat{\mathbf{p}}^H(i)\mathbf{T}(i)\bar{\mathbf{w}}(i) - v\right).$$

In the proposed JIO–NSG scheme, $\mathbf{T}(i)$ and $\bar{\mathbf{w}}(i)$ are jointly and iteratively computed. Let c denote the iteration number and define $c_{\max}$ as the total number of iterations for each time instant. We have $\mathbf{T}_0(i) = \mathbf{T}_{c_{\max}}(i-1)$ and $\bar{\mathbf{w}}_0(i) = \bar{\mathbf{w}}_{c_{\max}}(i-1)$. For the cth iteration, $\mathbf{T}_c(i)$ is updated with $\mathbf{T}_{c-1}(i)$ and $\bar{\mathbf{w}}_{c-1}(i)$ using (33), and then, $\bar{\mathbf{w}}_c(i)$ is trained with $\mathbf{T}_c(i)$ and $\bar{\mathbf{w}}_{c-1}(i)$ through (37).

Note that the complexity of the JIO–NSG scheme can be lower than the full-rank NSG algorithm, because there are several entries that are frequently reused in the update equations, e.g., the scalar term $\hat{\mathbf{p}}^H(i)\mathbf{r}(i)$, the vectors of $\mathbf{T}^H(i)\hat{\mathbf{p}}(i)$, and $\mathbf{T}^H(i)\mathbf{r}(i)$. However, the price that we pay for the complexity reduction is the requirement of extra storage space at the receiver.

B. BCE for the NSG Version

For the JIO–NSG receiver, we rearrange (24) as

$$\hat{\mathbf{V}}(i) = \mathbf{S}_e^H\mathbf{P}_r^H\hat{\mathbf{W}}(i) \quad (38)$$

where $\hat{\mathbf{W}}(i) = \mathbf{R}^{-m}(i)\mathbf{P}_r\mathbf{S}_e$. Here, we implement the Leakage SG algorithm to estimate $\hat{\mathbf{W}}(i)$, which can be expressed as [43]

$$\hat{\mathbf{W}}_l(i) = \lambda_v\hat{\mathbf{W}}_l(i-1) + \mu_v\left(\hat{\mathbf{W}}_{l-1}(i) - \mathbf{r}(i)\mathbf{r}^H(i)\hat{\mathbf{W}}_l(i-1)\right) \quad (39)$$

where $l = 1,\ldots,m$ is defined as the iteration index, λ_v is the leakage factor, and μ_v is the step size. Using (38), we obtain the leakage SG BCE, which is given by

$$\hat{\mathbf{h}}(i) = \hat{\mathbf{h}}(i-1) - \left(\hat{\mathbf{V}}(i)\hat{\mathbf{h}}(i-1)\right)/tr\left[\hat{\mathbf{V}}(i)\right]. \quad (40)$$

Finally, the effective signature vector of the desired user is given by

$$\hat{\mathbf{p}}(i) = \mathbf{P}_r\mathbf{S}_e\hat{\mathbf{h}}(i). \quad (41)$$

In terms of the computational complexity, we need $4mML$ multiplications and $3mML - mL$ additions for all the recursions in (39) and L^2M multiplications and $L^2M - L^2$ additions for (38).

TABLE I
ADAPTIVE VERSIONS OF THE PROPOSED JIO RECEIVER

NSG version:
Initialization: $\bar{\mathbf{w}}(1) = [1,1,1,\ldots,1]$, D-by-1 vector, $\mathbf{T}(1) = [\mathbf{I}_D \mid \mathbf{0}]^T$, M-by-D matrix. ($\mathbf{I}_D$ represents the D-by-D identity matrix.) *for* $i = 1,2,\ldots$ **1: Pre-adaptation:** $\bar{\mathbf{r}}(i) = \mathbf{T}^H(i)\mathbf{r}(i)$, $y(i) = \bar{\mathbf{w}}^H(i)\bar{\mathbf{r}}(i)$, Calculate $\hat{\mathbf{V}}(i)$ and $\hat{\mathbf{h}}(i)$ using (38) and (40), respectively, Calculate $\hat{\mathbf{p}}(i)$ using (41), Set $\mathbf{T}_0(i+1) = \mathbf{T}_{c_{max}}(i)$ and $\bar{\mathbf{w}}_0(i+1) = \bar{\mathbf{w}}_{c_{max}}(i)$. **2: Adaptation of $\mathbf{T}(i+1)$ and $\bar{\mathbf{w}}(i+1)$:** *for* $c = 1,2,\ldots,c_{max}$ Update $\mathbf{T}_c(i+1)$ using (33) with $\mathbf{T}_{c-1}(i+1)$ and $\bar{\mathbf{w}}_{c-1}(i+1)$, Update $\bar{\mathbf{w}}_c(i+1)$ using (37) with $\mathbf{T}_c(i+1)$ and $\bar{\mathbf{w}}_{c-1}(i+1)$, *end* Set $\mathbf{T}(i+1) = \mathbf{T}_{c_{max}}(i+1)$ and $\bar{\mathbf{w}}(i+1) = \bar{\mathbf{w}}_{c_{max}}(i+1)$ **4: Make Decision for the i-th data bit:** $\hat{b}(i) = \text{sign}(\Re(y(i)))$
RLS version:
Initialization: $\bar{\mathbf{w}}(1) = [1,1,1,\ldots,1]$, D-by-1 vector, $\mathbf{t}_d(1) = [1,0,0,\ldots,0]$ ($d = 1,2,\ldots,D$), D-by-1 vectors, $\bar{\mathbf{d}}(0) = [0,0,\ldots,0]$, D-by-1 vector, $\hat{\mathbf{R}}_y^{-1}(0) = \mathbf{I}_M/\delta$, M-by-M matrix, $\hat{\mathbf{R}}_{\mathrm{T}}^{-1}(0) = \mathbf{I}_D/\delta$, D-by-D matrix, ($\mathbf{I}_M$ is the M-by-M identity matrix. δ is a positive constant.) *for* $i = 1,2,\ldots$ **1: Pre-adaptation:** $\bar{\mathbf{r}}(i) = \mathbf{T}^H(i)\mathbf{r}(i)$, $y(i) = \bar{\mathbf{w}}^H(i)\bar{\mathbf{r}}(i)$, $\bar{\mathbf{d}}(i) = \bar{\mathbf{d}}(i-1) + \alpha\bar{\mathbf{r}}(i)y^*(i)$, Estimate $\hat{\mathbf{R}}_y^{-1}(i)$ and $\hat{\mathbf{R}}_{\mathrm{T}}^{-1}(i)$ using (46) and (52), respectively, Calculate $\hat{\mathbf{V}}(i)$ and $\hat{\mathbf{h}}(i)$ using (55) and (54), respectively, Calculate $\hat{\mathbf{p}}(i)$ using (56). **2: Adaptation of $\mathbf{t}_d(i)$:** *for* $d = 1,2,\cdots,D$ Calculate $\lambda_{t,d}(i)$ using (48), Update $\mathbf{t}_d(i)$ using (47), Normalize $\mathbf{t}_d(i) \leftarrow \mathbf{t}_d(i)/\parallel\mathbf{t}_d(i)\parallel$. **3: Adaptation of $\bar{\mathbf{w}}(i)$:** Calculate $\lambda_{Rw}(i)$ using (53), Update $\bar{\mathbf{w}}(i)$ using (51), **4: Make Decision for the i-th data bit:** $\hat{b}(i) = \text{sign}(\Re(y(i)))$

The JIO–NSG version is summarized in Table I.

V. PROPOSED JOINT ITERATIVE OPTIMIZATION–RECURSIVE LEAST SQUARES ALGORITHMS

In this section, we detail the RLS version of the proposed JIO scheme. In the JIO scheme, the $M \times D$ (where $D \ll M$)

transformation matrix can be expressed as

$$\mathbf{T}(i) = [\mathbf{t}_1(i), \mathbf{t}_2(i), \ldots, \mathbf{t}_D(i)]\,. \quad (42)$$

Note that the reduced-rank received signal can be expressed as $\bar{\mathbf{r}}(i) = \mathbf{T}^H(i)\mathbf{r}(i)$, whose dth element is $\bar{r}_d(i) = \mathbf{t}_d^H(i)\mathbf{r}(i)$. Because the transformation matrix projects the received signal onto a small-dimensional subspace, these vectors $\mathbf{t}_d(i)$ can be considered the direction vectors on each dimension of the subspace. For each time instant, we compute these M-dimensional vectors $\mathbf{t}_d(i)$ (where $d = 1, 2, \ldots, D$) one by one. One of the advantages of this process method in the RLS version is that the complexity of training the transformation matrix can be reduced with an approximation, which will be shown in the following discussion. In addition, this method provides a better representation of the transformation matrix and leads to better performance than the approach that updates all the columns of the projection matrix together. Note that the NSG version can also be modified to update the columns of the transformation matrix one by one, but the limited improved performance in NSG version is not worth the increased complexity.

After the projection, $\bar{\mathbf{r}}(i)$ is fed into the reduced-rank filter $\bar{\mathbf{w}}(i)$, and the output signal is given by

$$y(i) = \bar{\mathbf{w}}^H(i)\mathbf{T}^H(i)\mathbf{r}(i) = \bar{\mathbf{w}}^H(i)\sum_{d=1}^{D}\mathbf{t}_d^H(i)\mathbf{r}(i)\mathbf{q}_d$$

where $\mathbf{q}_d$ (where $d = 1, 2, \ldots, D$) are the vectors whose dth elements are ones, whereas all the other elements are zeros. In this section, an adaptive BCE is employed, and $\mathbf{t}_d(i)$ are jointly and iteratively optimized with $\bar{\mathbf{w}}(i)$ through RLS algorithms.

A. JIO–RLS Algorithms

In the JIO–RLS scheme, we need to solve the optimization problem

$$\begin{aligned}[\bar{\mathbf{w}}(i), \mathbf{t}_1(i), \ldots, \mathbf{t}_D(i)] &= \arg\min_{\mathbf{w}}(i), \mathbf{t}_1(i), \ldots, \mathbf{t}_D(i)\mathbf{J}_{\text{JIO-RLS}} \\ &\quad \times (\bar{\mathbf{w}}(i), \mathbf{t}_1(i), \ldots, \mathbf{t}_D(i))\end{aligned} \quad (43)$$

subject to the constraint $\bar{\mathbf{w}}^H(i)\sum_{d=1}^{D}\mathbf{t}_d^H(i)\hat{\mathbf{p}}(i)\mathbf{q}_d = v$, where $\hat{\mathbf{p}}(i)$ is the estimated signature vector that is obtained through BCE, which will be detailed in Section V-B. v is a real-valued constant for ensuring the convexity of the CM cost function

$$\mathbf{J}_{\text{JIO-RLS}}(\bar{\mathbf{w}}(i), \mathbf{t}_1(i), \ldots, \mathbf{t}_D(i)) = \frac{1}{2}\sum_{j=1}^{i}\alpha^{i-j}\left(|y(j)|^2 - 1\right)^2$$

where $0 < \alpha \leq 1$ is the forgetting factor, and $y(i)$ is the output signal at the ith time instant. Let us now consider the problem through the Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{JIO-RLS}}(\bar{\mathbf{w}}(i), \mathbf{t}_1(i), \ldots, \mathbf{t}_D(i)) &= \frac{1}{2}\sum_{j=1}^{i}\alpha^{i-j}\left(|y(j)|^2 - 1\right)^2 \\ &\quad + \Re\left[\lambda_R(i)\left(\bar{\mathbf{w}}^H(i)\sum_{d=1}^{D}\mathbf{t}_d^H(i)\hat{\mathbf{p}}(i)\mathbf{q}_d - v\right)\right]\end{aligned} \quad (44)$$

where $\lambda_R(i)$ is a complex-valued Lagrange multiplier. In the proposed JIO–RLS scheme, for each time instant, we first update the vectors $\mathbf{t}_d(i)$ (where $d = 1, 2, \ldots, D$) while assuming that $\bar{\mathbf{w}}(i)$ and other column vectors are fixed. Then, we adapt the reduced-rank filter with the updated transformation matrix.

For the update of the column vectors of the transformation matrix, we can express the output signal as follows:

$$y(i) = \bar{\mathbf{w}}^H(i)\sum_{d=1}^{D}\mathbf{t}_d^H(i)\mathbf{r}(i)\mathbf{q}_d = \bar{w}_d^*(i)\bar{r}_d(i) + \bar{\mathbf{w}}^H(i)\bar{\mathbf{r}}_e(i)$$

where the D-dimensional vector $\bar{\mathbf{r}}_e(i)$ can be obtained by calculating the reduced-rank received signal $\bar{\mathbf{r}}(i)$ and setting its dth element to zero. By taking the gradient terms of (44) with respect to $\mathbf{t}_d(i)$ and setting them to a null vector, we have $\nabla_{t_d}\mathcal{L}_{\text{JIO-RLS}} = \sum_{j=1}^{i}\alpha^{i-j}e(j)\mathbf{r}(j)(|\bar{w}_d(j)|^2\mathbf{r}^H(j)\mathbf{t}_d(i) + \bar{w}_d^*(j)\bar{\mathbf{r}}_e^H(j)\bar{\mathbf{w}}(j)) + (1/2)\lambda_{t,d}(i)\bar{w}_d^*(i)\hat{\mathbf{p}}(i) = 0$, where $e(i) = |y(i)|^2 - 1$, and $\lambda_{t,d}(i)$ is the complex-valued Lagrange multiplier to update the dth column vector in the transformation matrix. Rearranging the terms, we obtain

$$\mathbf{t}_d(i) = -\mathbf{R}_{\text{d}}^{-1}(i)\left(\frac{\lambda_{t,d}(i)}{2}\bar{w}_d^*(i)\hat{\mathbf{p}}(i) + \mathbf{v}_r(i)\right) \quad (45)$$

where we define the M-dimensional vector $\mathbf{v}_r(i) = \sum_{j=1}^{i}\alpha^{i-j}\bar{w}_d^*(j)\mathbf{r}(j)(e(j)\mathbf{r}_e^H(j)\bar{\mathbf{w}}(j) - \bar{w}_d(j)\bar{r}_d^*(j))$ and the $M \times M$ matrix $\mathbf{R}_{\text{d}}(i) = \sum_{j=1}^{i}\alpha^{i-j}|\bar{w}_d(j)|^2|y(j)|^2\mathbf{r}(j)\mathbf{r}^H(j)$. Note that $\mathbf{R}_{\text{d}}(i)$ is dependent on $\bar{w}_d(i)$, which is the dth element of the reduced-rank filter. Hence, to update each $\mathbf{t}_d(i)$, we need to calculate the corresponding $\mathbf{R}_{\text{d}}^{-1}(i)$, which leads to high computational complexity. In this paper, we devise an approximation $\mathbf{R}_{\text{d}}(i) \approx |\bar{w}_d(i)|^2\sum_{j=1}^{i}\alpha^{i-j}|y(j)|^2\mathbf{r}(j)\mathbf{r}^H(j) = |\bar{w}_d(i)|^2\mathbf{R}_y(i)$. Then, we adopt the matrix inversion lemma [44] to recursively estimate $\mathbf{R}_y^{-1}(i)$ as follows:

$$\begin{aligned}\boldsymbol{\kappa}_y(i) &= \hat{\mathbf{R}}_y^{-1}(i-1)y(i)\mathbf{r}(i) \\ \phi_y(i) &= \frac{1}{\alpha + y^*(i)\mathbf{r}^H(i)\boldsymbol{\kappa}_y(i)} \\ \hat{\mathbf{R}}_y^{-1}(i) &= \frac{1}{\alpha}\left(\hat{\mathbf{R}}_y^{-1}(i-1) - (\phi(i)\boldsymbol{\kappa}_y(i))\,\boldsymbol{\kappa}_y^H(i)\right)\end{aligned} \quad (46)$$

where $\hat{\mathbf{R}}_y^{-1}(i)$ is the estimate of $\mathbf{R}_y^{-1}(i)$. We use $\hat{\mathbf{R}}_y^{-1}(i)$ for all the adaptations of $\mathbf{t}_d(i)$ to avoid the estimation of the $\mathbf{R}_{\text{d}}^{-1}(i)$ (where $d = 1, 2, \ldots, D$), and the new update equation is given by

$$\mathbf{t}_d(i) = -\frac{\hat{\mathbf{R}}_y^{-1}(i)}{|\bar{w}_d(i)|^2}\left(\frac{\lambda_{t,d}(i)}{2}\bar{w}_d^*(i)\hat{\mathbf{p}}(i) + \mathbf{v}_r(i)\right). \quad (47)$$

Using the constraint $\bar{\mathbf{w}}^H(i)\sum_{d=1}^{D}\mathbf{t}_d^H(i)\hat{\mathbf{p}}(i)\mathbf{q}_d = v$, we obtain the expression of the Lagrange multiplier as

$$\lambda_{t,d}(i) = 2\left[\frac{\bar{w}_d^*(i)\mathbf{v}_r^H(i)\hat{\mathbf{R}}_y^{-1}(i)\hat{\mathbf{p}}(i) + \left(v - \bar{\mathbf{w}}^H(i)\hat{\mathbf{p}}_d(i)\right)|\bar{w}_d(i)|^2}{-|\bar{w}_d(i)|^2\,\hat{\mathbf{p}}^H(i)\hat{\mathbf{R}}_y^{-1}(i)\hat{\mathbf{p}}(i)}\right]^* \quad (48)$$

where $\hat{\mathbf{p}}_d(i)$ can be obtained by calculating the vector $\mathbf{T}^H(i)\hat{\mathbf{p}}(i)$ and setting its dth element to zero. Note that, in the update equation (47), small values of $|\bar{w}_d(i)|^2$ may cause numerical problems for later calculation. This issue can be addressed by normalizing the column vector after each adaptation, which is given by $\mathbf{t}_d(i) \leftarrow \mathbf{t}_d(i)/\|\mathbf{t}_d(i)\|$.

After updating the transformation matrix column by column, we will adapt the reduced-rank filter $\bar{\mathbf{w}}(i)$. By assuming that the transformation matrix is fixed, we can express the output signal in a simpler way as

$$y(i) = \bar{\mathbf{w}}^H(i)\mathbf{T}^H(i)\mathbf{r}(i) \tag{49}$$

where $\mathbf{T}(i) = [\mathbf{t}_1(i), \ldots, \mathbf{t}_D(i)]$, and the constraint can be expressed as $\bar{\mathbf{w}}^H(i)\mathbf{T}^H(i)\hat{\mathbf{p}}(i) = v$. Hence, the Lagrangian becomes

$$\mathcal{L}_{\mathrm{JIO-RLS}}\left(\bar{\mathbf{w}}(i), \mathbf{T}(i)\right) = \frac{1}{2}\sum_{j=1}^{i}\alpha^{i-j}\left(|y(j)|^2 - 1\right)^2 + \Re\left[\lambda_R(i)\left(\bar{\mathbf{w}}^H(i)\mathbf{T}^H(i)\hat{\mathbf{p}}(i) - v\right)\right]. \tag{50}$$

By taking the gradient terms of (50) with respect to $\bar{\mathbf{w}}(i)$ and setting them to a null vector, we have $\nabla_{\mathbf{w}}\mathcal{L}_{\mathrm{JIO-RLS}} = \sum_{j=1}^{i}\alpha^{i-j}e(j)\mathbf{T}^H(j)\mathbf{r}(j)\mathbf{r}^H(j)\mathbf{T}(j)\bar{\mathbf{w}}(i) + (1/2)\lambda_{Rw}(i) \times \mathbf{T}^H(i)\hat{\mathbf{p}}(i) = \mathbf{0}$, where the real-valued error is $e(i) = (|y(i)|^2 - 1)$, and $\lambda_{Rw}(i)$ is the complex-valued Lagrange multiplier to update the reduced-rank filter. Rearranging the terms, we obtain

$$\bar{\mathbf{w}}(i) = \mathbf{R}_{\mathrm{T}}^{-1}(i)\left(-\frac{\lambda_{Rw}(i)}{2}\mathbf{T}^H(i)\hat{\mathbf{p}}(i) + \bar{\mathbf{d}}(i)\right) \tag{51}$$

where $\mathbf{R}_{\mathrm{T}}(i) = \sum_{j=1}^{i}\alpha^{i-j}|y(j)|^2\bar{\mathbf{r}}(j)\bar{\mathbf{r}}^H(j)$, and $\bar{\mathbf{d}}(i) = \sum_{j=1}^{i}\alpha^{i-j}\bar{\mathbf{r}}(j)y^*(j) = \bar{\mathbf{d}}(i-1) + \alpha\bar{\mathbf{r}}(i)y^*(i)$. The matrix inversion lemma [44] is again used to recursively estimate the inversion matrix $\mathbf{R}_{\mathrm{T}}^{-1}(i)$ as follows:

$$\begin{aligned}
\boldsymbol{\kappa}_{\mathrm{T}}(i) &= \hat{\mathbf{R}}_{\mathrm{T}}^{-1}(i-1)\bar{\mathbf{r}}(i)y(i)\\
\phi_{\mathrm{T}}(i) &= \frac{1}{\alpha + y(i)^*\bar{\mathbf{r}}^H(i)\boldsymbol{\kappa}_{\mathrm{T}}(i)}\\
\hat{\mathbf{R}}_{\mathrm{T}}^{-1}(i) &= \frac{1}{\alpha}\left(\hat{\mathbf{R}}_{\mathrm{T}}^{-1}(i-1) - \left(\phi_{\mathrm{T}}(i)\boldsymbol{\kappa}_{\mathrm{T}}(i)\right)\boldsymbol{\kappa}_{\mathrm{T}}^H(i)\right)
\end{aligned} \tag{52}$$

where $\hat{\mathbf{R}}_{\mathrm{T}}^{-1}(i)$ is the estimate of $\mathbf{R}_{\mathrm{T}}^{-1}(i)$. To calculate the Lagrange multiplier, we use the constraint $\bar{\mathbf{w}}^H(i)\mathbf{T}^H(i)\hat{\mathbf{p}}(i) = v$ and obtain

$$\lambda_{Rw}(i) = 2\left[\frac{\bar{\mathbf{d}}^H(i)\mathbf{R}_{\mathrm{T}}^{-1}(i)\mathbf{T}^H(i)\hat{\mathbf{p}}(i) - v}{\hat{\mathbf{p}}^H(i)\mathbf{T}(i)\mathbf{R}_{\mathrm{T}}^{-1}(i)\mathbf{T}^H(i)\hat{\mathbf{p}}(i)}\right]^*. \tag{53}$$

B. BCE for the RLS Version

In the JIO–RLS algorithm, the estimation of the covariance matrix $\mathbf{R}_y(i) = \sum_{j=1}^{i}\alpha^{i-j}|y(j)|^2\mathbf{r}(j)\mathbf{r}^H(j)$ and its inversion are obtained in the stage of adapting the transformation matrix. Note that $|y(j)|^2$ tends to 1 as the number of received signal increases. Hence, by replacing the inverse matrix $\mathbf{R}^{-1}(i)$ in (24) with $\mathbf{R}_y^{-1}(i)$, we obtain

$$\hat{\mathbf{h}}(i) = \hat{\mathbf{h}}(i-1) - \left(\hat{\mathbf{V}}(i)\hat{\mathbf{h}}(i-1)\right)/tr\left[\hat{\mathbf{V}}(i)\right] \tag{54}$$

where the $L \times L$ matrix is defined as

$$\hat{\mathbf{V}}(i) = \mathbf{S}_e^H\mathbf{P}_r^H\mathbf{R}_y^{-m}(i)\mathbf{P}_r\mathbf{S}_e \tag{55}$$

and the effective signature vector of the desired user is given by

$$\hat{\mathbf{p}}(i) = \mathbf{P}_r\mathbf{S}_e\hat{\mathbf{h}}(i). \tag{56}$$

Using $\mathbf{R}_y^{-1}(i)$ instead of $\mathbf{R}^{-1}(i)$ can save $\mathcal{O}(M^2)$ computational complexity for the JIO–RLS version, and simulation results will demonstrate that the performance will not be degraded with this replacement. The JIO–RLS version is summarized in Table I.

VI. Complexity Analysis and Rank Adaptation Algorithm

In this section, a complexity analysis is presented to compare the two versions of the JIO receiver, the full-rank NSG and RLS schemes, and the NSG and RLS versions of the MSWF. The computational complexity of the BCEs that are implemented in this paper is also analyzed. A rank adaptation algorithm is detailed in this section, which can adaptively select the rank and achieve better tradeoffs between the convergence speed and the steady-state performances.

A. Complexity Analysis

As shown in Table II, the complexity of the analyzed blind CCM full-rank NSG and RLS, MSWF-NSG and MSWF-RLS [12], and the proposed NSG and RLS versions of the JIO scheme is compared with respect to the number of complex additions and complex multiplications for each time instant. The complexity of the conventional BCE that is described in Section III is compared with the BCEs for the JIO–NSG and JIO–RLS that are described in Sections IV-B and V-B, respectively.

For the analysis of the adaptive algorithms, the quantity M is the length of the full-rank filter, D is the dimension of reduced-rank filter, and $c_{\max}$ is the number of iterations for the JIO–NSG version in each time instant. Note that only one iteration is required in the JIO–RLS version for each time instant. For the analysis of the BCEs, the quantity L is the length of the CIR, and m is the power of the inverse covariance matrix. In this paper, M is the minimum integer that is larger than the scalar term $(T_s/T_\tau + T_{DS}/T_\tau - 1)/(T_c/T_\tau) = (T_s + T_{DS} - T_\tau)/T_c$, and $L = T_{DS}/T_\tau$. Because T_τ is set to 0.125 ns as in the standard IEEE802.15.4a channel model, the symbol T_s and chip T_c durations are assumed given for the designer. Hence, M and L are both related to the channel delay spread T_{DS}. In this paper, the parameters are set as follows: $T_s = 12$ ns, $T_c = 0.375$ ns, $m = 3$, and $c_{\max} = 3$. As shown in Fig. 2, the number of complex multiplications required for different algorithms are compared as a function of the channel

TABLE II
COMPLEXITY ANALYSIS

	Complex Additions	Complex Multiplications
Full-Rank NSG	M^2+3M-1	$2M^2+4M+5$
Full-Rank RLS	$5M^2+2M+1$	$5M^2+3M+1$
MSWF-NSG	$DM^2+(2D+2)M-2D^2-2$	$(D+1)M^2+(4D+2)M-2D^2+4D+5$
MSWF-RLS	$DM^2+(2D+2)M+2D^2-D$	$(D+1)M^2+(4D+2)M+2D^2+3D+1$
JIO-NSG	$c_{max}(6DM+3M+4D-2)$	$c_{max}(8DM+4M+7D+11)$
JIO-RLS	$DM^2+3DM+4D^2-4D$	$DM^2+6DM+4D^2+15D+1$
Conventional BCE	$(m+1)M^2L-(m+1)ML+2M^2+L^2+L-1$	$(m+1)M^2L+3M^2+L^2+2M+1$
BCE for JIO-NSG	$L^2M+3mML-(m-1)L-1$	$L^2M+4mML+L^2$
BCE for JIO-RLS	$(m+1)M^2L-(m+1)ML+L^2+L-1$	$(m+1)M^2L+L^2$

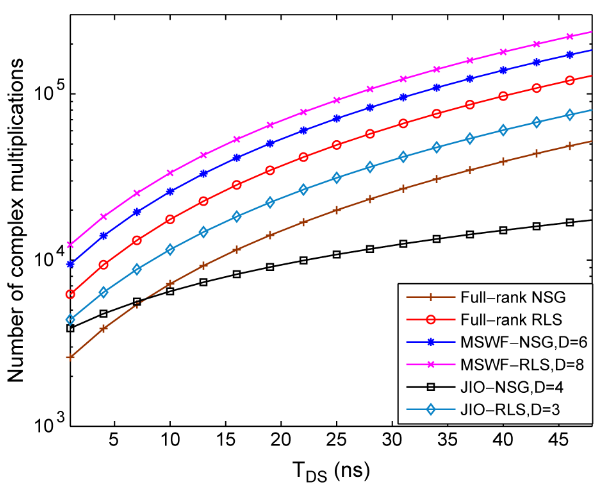

Fig. 2. Number of multiplications that are required for different algorithms.

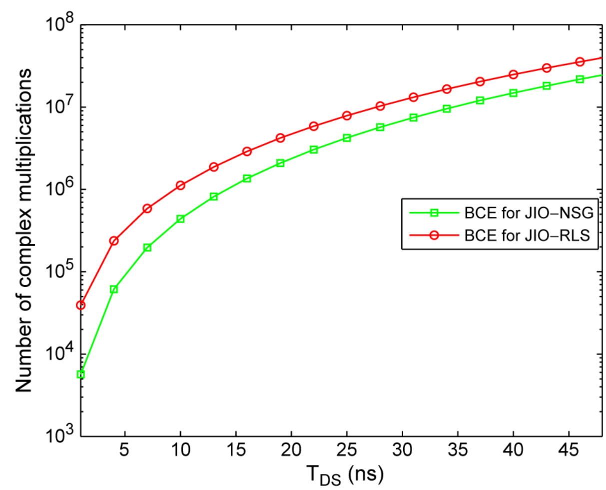

Fig. 3. Number of multiplications that are required for BCEs.

delay spread T_{DS}. The JIO–RLS algorithm with $D=3$ has lower complexity than the MSWF algorithms and the full-rank RLS. It will be demonstrated by the simulation results that the JIO–RLS algorithm can achieve fast convergence with a very small rank $(D<5)$. The proposed JIO–NSG algorithm has lower complexity than the full-rank NSG algorithm in the long channel delay spread scenarios. As discussed in Section IV-A, the price that we pay for such a complexity reduction is the extra storage space at the receiver.

The complexity of the BCEs for the JIO versions is shown in Fig. 3, in which the number of complex multiplications is shown as a function of channel delay spread T_{DS}. The complexity of the BCE for the JIO–NSG version has lower complexity than the BCE for the JIO–RLS version in all the analyzed scenarios.

B. Rank Adaptation

In the proposed blind JIO reduced-rank receiver, the computational complexity and the performance are sensitive to the determined rank D. In this section, a rank adaptation algorithm is employed to achieve better tradeoffs between the performance and the complexity of the JIO receiver. The rank adaptation algorithm is based on the *a posteriori* LS cost function to estimate the MSE, which is a function of $\bar{\mathbf{w}}_D(i)$ and $\mathbf{T}_D(i)$ and can be expressed as

$$\mathcal{C}_D(i)=\sum_{n=0}^{i}\lambda_D^{i-n}\left(\left|\bar{\mathbf{w}}_D^H(n)\mathbf{T}_D^H(n)\mathbf{r}(n)\right|^2-1\right)^2 \quad (57)$$

where λ_D is a forgetting factor. Because the optimal rank can be considered a function of the time interval i [19], the forgetting factor is required and allows us to track the optimal rank. For each time instant, we update a transformation matrix $\mathbf{T}_{\mathrm{M}}(i)$ and a reduced-rank filter $\bar{\mathbf{w}}_{\mathrm{M}}(i)$ with the maximum rank $D_{\max}$, which can be expressed as

$$\begin{aligned}\mathbf{T}_{\mathrm{M}}(i)&=\left[\mathbf{t}_{\mathrm{M},1}(i),\ldots,\mathbf{t}_{\mathrm{M},D}(i),\ldots,\mathbf{t}_{\mathrm{M},D_{\max}}(i)\right]^T\\ \bar{\mathbf{w}}_{\mathrm{M}}(i)&=\left[\bar{w}_{\mathrm{M},1}(i),\ldots,\bar{w}_{\mathrm{M},D}(i),\ldots,\bar{w}_{\mathrm{M},D_{\max}}(i)\right]^T. \end{aligned}\quad (58)$$

After the adaptation, we test values of D within the range $D_{\min}$ to $D_{\max}$. For each tested rank, we use the following estimators:

$$\begin{aligned}\mathbf{T}_D(i)&=\left[\mathbf{t}_{\mathrm{M},1}(i),\ldots,\mathbf{t}_{\mathrm{M},D}(i)\right]^T\\ \bar{\mathbf{w}}_D(i)&=\left[\bar{w}_{\mathrm{M},1}(i),\ldots,\bar{w}_{\mathrm{M},D}(i)\right]^T\end{aligned}\quad (59)$$

and substitute (59) into (57) to obtain the value of $\mathcal{C}_D(i)$, where $D\in\{D_{\min},\ldots,D_{\max}\}$. The proposed algorithm can be expressed as

$$D_{\mathrm{opt}}(i) = \arg \min_{D \in \{D_{\min}, \ldots, D_{\max}\}} \mathcal{C}_D(i). \tag{60}$$

We remark that the complexity of updating the reduced-rank filter and the transformation matrix in the proposed rank adaptation algorithm is the same as the receiver with rank $D_{\max}$, because we only adapt the $\mathbf{T}_{\mathrm{M}}(i)$ and $\bar{\mathbf{w}}_{\mathrm{M}}(i)$ for each time instant. However, additional computations are required for calculating the values of $\mathcal{C}_D(i)$ and selecting the minimum value of a $(D_{\max} - D_{\min} + 1)$-dimensional vector that corresponds to a simple search and comparison.

VII. Simulations

In this section, the proposed NSG and RLS versions of the blind JIO adaptive receivers are applied to the uplink of a multiuser BPSK DS-UWB system. The performance of the proposed receivers are compared with the RAKE receiver with the maximal-ratio combining (MRC), the NSG and RLS versions of the full-rank schemes, and the MSWF. Note that the BCE described in Section III is implemented to provide channel coefficients to the RAKE receiver, and its bit-error-rate (BER) performance is averaged for comparison. In all simulations, all the users are assumed to continuously transmit at the same power level. The pulse shape that is adopted is the RRC pulse with the a pulsewidth of 0.375 ns. The spreading codes are randomly generated for each user, with a spreading gain of 32, and the data rate of the communication is approximately 83 Mbps. We assess the blind receivers with the standard IEEE 802.15.4a channel models of channel model 1 (ChMo1) and channel model 2 (ChMo2), which are for indoor residential line-of-sight (LOS) and non-line-of-sight (NLOS) environments, respectively [41]. We assume that the channel is constant during the whole transmission. The channel delay spread is $T_{DS} = 10$ ns, and the ISI from two neighbor symbols are taken into account for the simulations. The sampling rate at the receiver is assumed to be 2.67 GHz, and the length of the discrete time received signal is $\mathrm{M} = 59$. For all the experiments, all the adaptive receivers are initialized as vectors, with all the elements equal to 1. This approach allows fair comparison between the analyzed techniques for their convergence performance. In practice, the filters can be initialized with prior knowledge about the spreading code or the channel to accelerate the convergence. In all the simulations, the phase of $h(0)$ is used as a reference to remove the phase ambiguity derived from the BCEs. All the curves shown in this section are obtained by averaging 200 independent runs. In this section, the coded BER performance is obtained by adopting a convolutional code with a coding rate of 2/3. The code polynomial is [7,5,5], and the constraint length is set to 5. Note that other coding schemes that employ turbo codes, low-density parity-check (LDPC) codes, and/or iterative detection [45] can be employed to further improve the performance of the proposed algorithms.

First, we assess the mean square error (MSE) performance of the BCE that is introduced in Section III with ChMo2. As shown in Fig. 4, the MSE performance is shown as a function of the number of transmitted symbols with different channel delays in a sever-user scenario with a signal-to-noise ratio (SNR) of 20 dB. The performance of the BCE is highly dependent on the channel delays. The performance of the CCM-based blind adaptive algorithms will significantly be degraded in the scenarios of large channel delays due to the inaccuracy of the BCE. In this paper, we consider a channel delay of 10 ns.

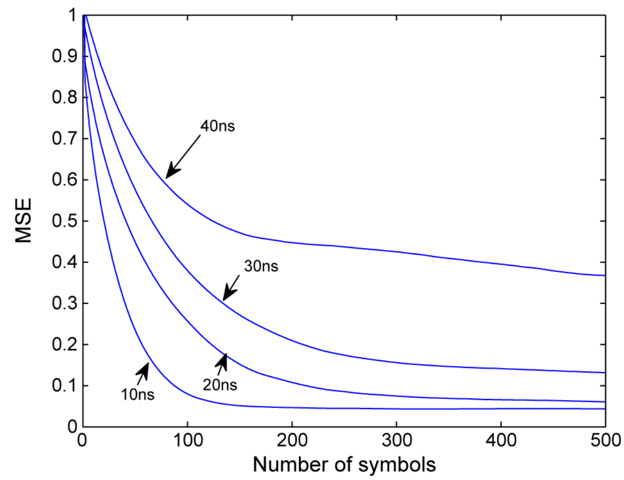

Fig. 4. MSE performance of the blind channel estimation (with ChMo2).

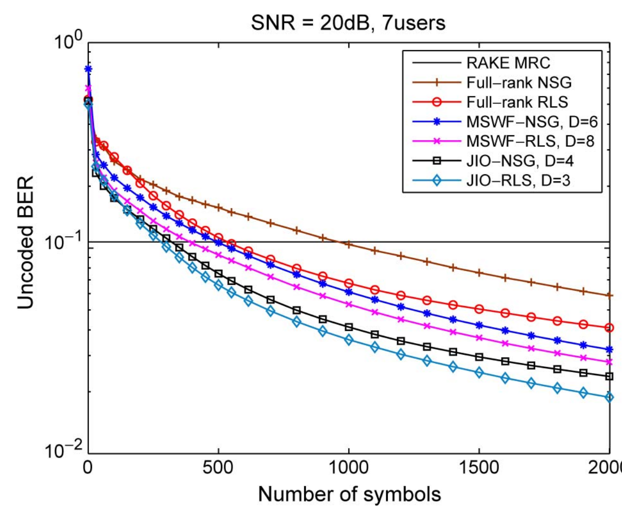

Fig. 5. Uncoded BER performance of different algorithms with ChMo2. For the full-rank NSG, $\mu = 0.025$. For the full-rank RLS, $\delta = 10$, and $\lambda = 0.9998$. For MSWF-NSG, $D = 6$, and $\mu = 0.025$. For MSWF-RLS, $D = 8$, and $\lambda = 0.998$. For JIO-NSG, $D = 4$, $c_{\max} = 3$, $\upsilon = 1$, $\mu_{T,0} = 0.075$, and $\mu_{w,0} = 0.005$. For JIO-RLS, $D = 3$, $\lambda = 0.9998$, $\delta = 10$, and $\upsilon = 0.5$.

In Fig. 5, we compare the uncoded BER performance of the proposed JIO receivers with the full-rank NSG and RLS algorithms, MSWF-NSG, and MSWF-RLS in the NLOS environment (ChMo2). In a seven-user scenario with an SNR of 20 dB, the uncoded BER performance of different algorithms as a function of symbols transmitted is presented, which enables us to compare the convergence rate of different adaptive algorithms. Among all the analyzed algorithms, the proposed JIO–RLS algorithm is the fastest approach. The JIO–NSG algorithm outperforms the MSWF versions and the full-rank versions with a low complexity. A noticeable improvement on the BER performance is obtained by using the JIO receivers.

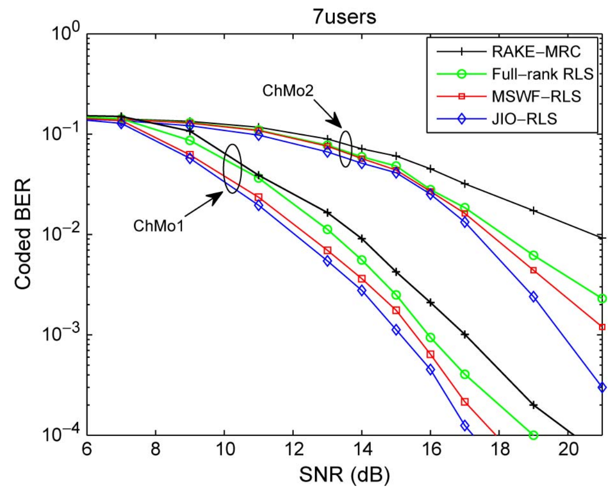

Fig. 6. BER performance of the proposed scheme with different SNRs.

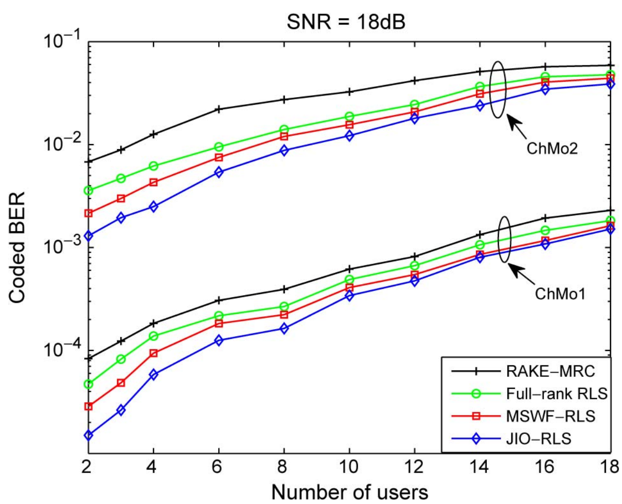

Fig. 7. BER performance of the proposed scheme with different numbers of users.

In Figs. 6 and 7, we assess the coded BER performances of the blind algorithms with different SNRs in a seven-user scenario and with different numbers of users in an 18-dB SNR scenario, respectively. Both ChMo1 and ChMo2 are considered in these simulations. The parameters that are set for all the adaptive algorithms are the same, as shown in Fig. 5. The proposed JIO versions show better MAI and ISI canceling capability in all the simulated scenarios. It can be observed that the use of channel coding improves the performance of the receivers and the same hierarchy in terms of BER performance is verified—the proposed JIO–RLS algorithm achieves the best performance. In Fig. 6, the JIO–RLS can save around 2 dB compared with the MSWF–RLS with ChMo2 for a BER of around 10^{-3} and save around 1 dB with ChMo1 for a BER of around 10^{-4}. In Fig. 7, the JIO–RLS scheme can support more than two additional users compared with the MSWF–RLS with ChMo2 for a BER of around 10^{-3} and can support more than one additional user with ChMo1 for a BER of around 10^{-4}.

In Fig. 8(a), the SINR performance is shown as a function of the rank D in the NLOS environment (ChMo2). We consider a seven-user scenario with an SNR of 20 dB. A noticeably better

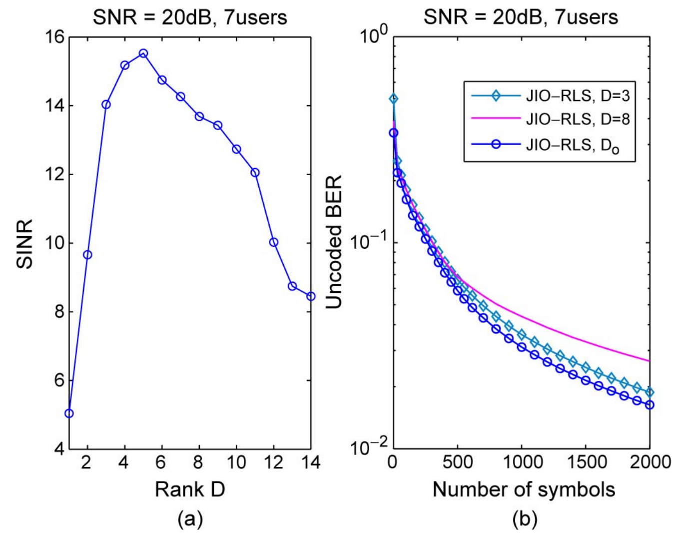

Fig. 8. (a) SINR performance with different ranks D (with ChMo2). (b) Uncoded BER performance of the rank-adaptation algorithm (with ChMo2).

performance is obtained for the ranks in the range of 3–8. In this scenario, $\mathrm{D}=5$ performs best, and a 1.5-dB gain is achieved compared with the algorithm with $\mathrm{D}=3$ and $\mathrm{D}=8$. Note that, for the JIO–RLS algorithm, the complexity is $O(DM^2)$. The designer can choose the rank D as a parameter that will affect the complexity and the performance. Fig. 8(b) compares the uncoded BER performance in the NLOS environment (ChMo2) of the JIO–RLS using the rank adaptation algorithm given by (60) with $D_{\max}=8$ and $D_{\min}=3$. Results that use a fixed rank of 3 and 8 are also shown for comparison and the illustration of the sensitivity of the JIO scheme to the rank D. The forgetting factor is $\lambda_D=0.998$. It is shown that the uncoded BER performance of the JIO–RLS scheme with the rank adaptation algorithm outperforms the fixed-rank scenarios with $D_{\min}=3$ and $D_{\max}=8$. In this experiment, $D=3$ has better steady-state performance than $D=8$, with both cases showing similar convergence speed. The rank adaptation algorithm provides a better solution than the fixed-rank approaches. Note that the complexity of updating the transformation matrix and the reduced-rank filter in the rank adaptation algorithm is the same as the fixed-rank case with $D=D_{\max}$. Additional complexity is required to compute the values of $\mathcal{C}_D(i)$ by using (57) and selecting the minimum value of a $(D_{\max}-D_{\min}+1)$-dimensional vector.

In the last experiment, we examine the blind adaptive algorithms with an additional narrowband interference (NBI), which is modeled as a single-tone signal (complex baseband) [46], i.e.,

$$J(t)=\sqrt{P_j}e^{(2\pi f_d t+\theta_j)} \tag{61}$$

where P_j is the NBI power, f_d is the frequency difference between the carrier frequency of the UWB signal and of the NBI, and θ_j is the random phase, which is uniformly distributed in $[0,\pi)$. Here, the received signal can be expressed as

$$z(t)=\sum_{k=1}^{K}\sum_{l=0}^{L-1}h_{k,l}x^{(k)}(t-lT_\tau)+n(t)+J(t). \tag{62}$$

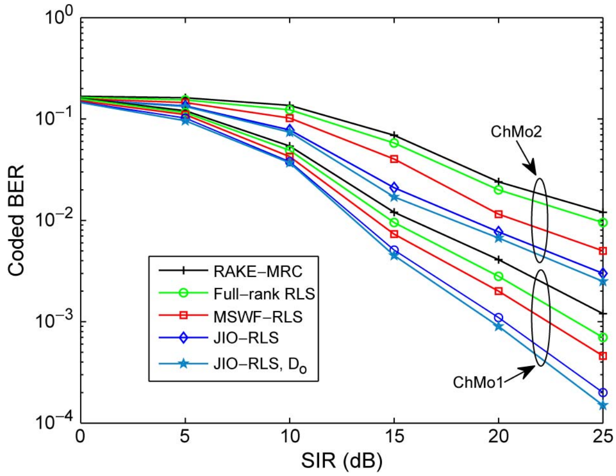

Fig. 9. BER performance of the adaptive algorithm with NBI, where $f_d =$ 23 MHz.

Note that, in this experiment, the receivers are required to blindly suppress the ISI, MAI, and NBI together. In Fig. 9, in a five-user system with an 18-dB SNR, the coded BER performance of the RLS versions are compared with different signal-to-NBI ratios (SIRs) with ChMo1 and ChMo2. The algorithms are set with the same parameters as in Fig. 5. With the NBI, the eigenvalue spread of the covariance matrix of the received signal is increased, and this change slows down the convergence rate of the full-rank scheme. However, the proposed JIO receiver shows better ability to cope with this change, and the performance gain over the full-rank scheme is increased compared to the NBI free scenarios. By adopting the rank adaptation algorithm, the performance is improved compared with the fixed-rank JIO–RLS receiver in the simulated scenarios. This case is mainly because of the faster convergence speed that is introduced by the rank adaptation algorithm.

VIII. Conclusion

A novel blind reduced-rank receiver has been proposed based on JIO and the CCM criterion. The novel receiver consists of a transformation matrix and a reduced-rank filter. The NSG and RLS adaptive algorithms are developed to update its parameters. In DS-UWB systems, both versions (NSG and RLS) of the proposed blind reduced-rank receivers outperform the analyzed CCM based full-rank and the existing reduced-rank adaptive schemes with low complexity. The robustness of the proposed receivers has been demonstrated in the scenario where the blind receivers are required to suppress the ISI, MAI, and NBI together. The proposed blind receivers can be employed in spread-spectrum systems that encounter large filter problems and suffer from severe interferences.

Appendix
Convergence Properties

In this section, we examine the convergence properties of the cost function $\mathbf{J}_{\text{JIO}} = (1/2)E[(|y(i)|^2-1)^2]$, where $y(i) = \bar{\mathbf{w}}^H(i)\mathbf{T}^H(i)\mathbf{r}(i)$. For simplicity, we drop the time index (i). The received signal is given by

$$\mathbf{r} = \sum_{k=1}^{K}\sqrt{E_k}\mathbf{P}_r\mathbf{S}_{e,k}\mathbf{h}_k b_k + \boldsymbol{\eta} + \mathbf{n}$$
$$= \sum_{k=1}^{K}\sqrt{E_k}b_k\mathbf{p}_k + \boldsymbol{\eta} + \mathbf{n} = \mathbf{P}_k\mathbf{A}_k\mathbf{b} + \boldsymbol{\eta} + \mathbf{n} \quad (63)$$

where $\mathbf{p}_k = \mathbf{P}_r\mathbf{S}_{e,k}\mathbf{h}_k$, $k = 1,\ldots,K$ are the signature vectors of the users. $\mathbf{P}_k = [\mathbf{p}_1,\ldots,\mathbf{p}_K]$, $\mathbf{A}_k = \text{diag}(\sqrt{E_1},\ldots,\sqrt{E_K})$, and $\mathbf{b} = [b_1,\ldots,b_K]$. $\boldsymbol{\eta}$ and $\mathbf{n}$ represent the ISI and AWGN, respectively. We assume that b_k, $k = 1,\ldots,K$, are statistically independent and identically distributed (i.i.d.) random variables with zero mean and unit variance and are independent to noise. First, we will discuss the noise-free scenario for the analysis, in which the output signal of the JIO receiver is given by

$$y = \bar{\mathbf{w}}^H\mathbf{T}^H\mathbf{P}_k\mathbf{A}_k\mathbf{b} = \boldsymbol{\epsilon}^H\mathbf{b} \quad (64)$$

where $\boldsymbol{\epsilon} \triangleq \mathbf{A}_k^H\mathbf{P}_k^H\mathbf{T}\bar{\mathbf{w}} = [\epsilon_1,\ldots,\epsilon_K]$. Assuming that user 1 is the desired user and recalling the constraint $\bar{\mathbf{w}}^H\mathbf{T}^H\mathbf{p}_1 = v$, where v is a real-valued constant, we obtain that the first element of the vector $\boldsymbol{\epsilon}$ can be expressed as

$$\epsilon_1 = \sqrt{E_1}\mathbf{p}_1^H\mathbf{T}\bar{\mathbf{w}} = \sqrt{E_1}v. \quad (65)$$

Now, let us have a closer look at the cost function

$$\mathbf{J}_{\text{JIO}} = \frac{1}{2}E\left[|y(i)|^4 - 2|y(i)|^2 + 1\right]$$
$$= \frac{1}{2}\left(E\left[(\boldsymbol{\epsilon}^H\mathbf{b}\mathbf{b}^H\boldsymbol{\epsilon})^2\right] - 2E\left[\boldsymbol{\epsilon}^H\mathbf{b}\mathbf{b}^H\boldsymbol{\epsilon}\right] + 1\right)$$
$$= \frac{1}{2}\left(\sum_{k=1}^{K}\sum_{j=1}^{K}|\epsilon_k|^2|\epsilon_j|^2|b_k|^2|b_j|^2 - 2\sum_{k=1}^{K}|\epsilon_k|^2|b_k|^2 + 1\right)$$
$$= \frac{1}{2}\left(\sum_{k=1}^{K}\sum_{j=1}^{K}|\epsilon_k|^2|\epsilon_j|^2 - 2\sum_{k=1}^{K}|\epsilon_k|^2 + 1\right)$$
$$= \frac{1}{2}\left(|\epsilon_1|^2 + \tilde{\boldsymbol{\epsilon}}^H\tilde{\epsilon}\right)^2 - \left(|\epsilon_1|^2 + \tilde{\boldsymbol{\epsilon}}^H\tilde{\boldsymbol{\epsilon}}\right) + \frac{1}{2} \quad (66)$$

where $\tilde{\boldsymbol{\epsilon}} = [\epsilon_2,\ldots,\epsilon_k] = \tilde{\mathbf{A}}_k^H\tilde{\mathbf{P}}_k^H\mathbf{T}\bar{\mathbf{w}}$, $\tilde{\mathbf{P}}_k = [\mathbf{p}_2,\ldots,\mathbf{p}_K]$, and $\tilde{\mathbf{A}}_k = \text{diag}(\sqrt{E_2},\ldots,\sqrt{E_K})$. Equation (66) transforms the cost function of both $\mathbf{T}$ and $\bar{\mathbf{w}}$ into a function with a single variable $\tilde{\boldsymbol{\epsilon}}$. Note that $\tilde{\boldsymbol{\epsilon}}$ is a linear function of $\mathbf{T}\bar{\mathbf{w}}$, which is the blind reduced-rank receiver. Hence, the convexity properties of the cost function with respect to $\tilde{\boldsymbol{\epsilon}}$ reflects the convexity properties of the cost function with respect to $\mathbf{T}\bar{\mathbf{w}}$. To evaluate the convexity of $\mathbf{J}_{\text{JIO}}$, we compute its Hessian, which is given by

$$\mathbf{H}_{JIO} = \frac{\partial}{\partial\tilde{\boldsymbol{\epsilon}}^H}\frac{\partial\mathbf{J}_{\text{JIO}}}{\partial\tilde{\boldsymbol{\epsilon}}} = 2\tilde{\boldsymbol{\epsilon}}\tilde{\boldsymbol{\epsilon}}^H + \left(|\epsilon_1|^2 - 1\right)\mathbf{I}. \quad (67)$$

It can be concluded that one sufficient condition for $\mathbf{H}_{JIO}$ to be a positive-definite matrix is $|\epsilon_1|^2 > 1$, which is $E_1v^2 > 1$. This condition is obtained in a noiseless scenario; however, it also holds for a small σ^2, which can be considered to be a slight perturbation of the noise-free case [10]. For larger values of σ^2, the term v can be adjusted to ensure the convexity of the cost function.

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