**Stochastic Processes**

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List 4

1. Compute the mean and the variance of the following random variables:

a) z, where ;

b) v, where ;

c) t, where

2. Consider a random variable with probability density function

and y another random variable which depends on x through the relation

Determine for the random variable y:

a) The mean;

b) The mean-square value;

c) The variance.

3. A traffic light is open for 30 seconds and closed for 30 seconds. This continues in an alternating fashion.

a) Determine the probability density function of the random variable that caracterizes the waiting time of a driver to get through the traffic light.

b) Determine the average waiting time.

4. Consider a random variable z with probability density function given by

Consider another two random variables x and y, defined by x = z2 and y=z3.

a) Are x and y uncorrelated? Please explain.

b) Show that x and y are not statistically independent.

5. The lifetime of an equipment is a random variable x with exponential probability density function with parameter a, that is,

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When the equipment fails, the required time to repair it can be modelled by a random variable y with probability density function given by

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Consider z the total duration of a cycle of the equipment. The elapsed time is the time measured from the instant when the equipment starts its operation to the time it is back to operation after a failure, that is,

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a) Determine the correlation coefficient between the random variables x and z.

b) Determine the probability density function of the random variable z.

c) Compute, for a = 1 and b =5, the probability that the duration of the cycle exceeds 10.

6. A random variable x has moment generating function given by

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a) Determine the value of k.

b) Compute the expected value of x.

7. A random variable x has moment generating function given by

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a) Determine the value of the constant k.

b) Compute the expected value of x.

c) If y = x + m then determine with m as a constant.

8. Consider the statistically independent and identically distributed random variables {x1, x2, ..., xn}. Each random variable has exponential probability density function with parameter 1 as described by

Consider also n a discrete random variable that is statistically independente of , with probability density function given by

Define the random variable described by

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where y=0 if n=0. Determine:

a) the moment generating function ;

b) the mean of thr random variable y;

c) the variance of the random variable y.