**Stochastic Processes**

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List 3

1. Determine the probability density function (or probability mass function) and the cumulative distribution function of a random variable described by

for the following cases:

a) x is a uniform random variable in the interval [-1, 1];

b) x is a Poisson random variable with parameter .

2. Consider a random variable x whose cumulative distribution function is . This random variable is applied to a device that transforms it according to the function , resulting in the random variable y.

a) Calculate the probability density function of the random variable y.

b) Suppose that we wish to obtain the random variable z with probability density function of Laplace described by

through the transformation given by

,

where y is a uniform random variable in the interval [0, 1]. Use the result of the previous item to specify a convenient function g(.).

3. The gain of a power amplifier is a random variable y with probability density function given by

The same gain measured in decibels is described by

Determine the probability density function of the random variable x.

4. Consider the random variables z and w, defined by

and ,

where x and y are random variables with joint probability density function described by

.

Determine the joint probability density function of the random variables z and w.

5. A device that is often used in communication and signal processing systems is known as the multiplier, as illustrated below. Suppose that two random signals x(t) and y(t) are sampled and the resulting samples are combined to form the samples of the signal z(t) as given by

Consider that each sample of the signal x(t) is a random variable that can take on the values +1 and -1 with probability 1/2. Consider also that each sample y of the signal y(t) is a Gaussian random variable with probability density function given by

.

a) Determine the probability density function of the random variable z that characterizes each sample of the signal z(t).

b) Are the random variables x and z statistically independent? Please explain.