**Stochastic Processes**

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List 2

1. Consider the throw of 2 dice and the experiment whose result consists of the sum of the number of points in the faces turned up of the dice.

a) Define this sum as a random variable.

b) Sketch the cumulative distribution function.

c) Compute the probability of x to take on values in the interval [7 , 9].

2. Consider the lifetime of a type of lamp in hours. This lifetime can be modeled by a random variable t with probability density function given by

$$p\_{t}\left(T\right)=ae^{-aT}u\left(T\right).$$

a) Determine and sketch the cumulative density function of the random variable t.

b) Suppose that the probability that the lifetime of the lamp exceeds 200 hours is $e^{-1}$, compute the value $T\_{0}$ such that the probability that the lifetime of the lamp is less than a $T\_{0}$ is 0,1.

3. Show that if x is a random variable with an exponential probability density function given by

$$p\_{t}\left(T\right)=ae^{-aT}u\left(T\right),$$

then we have

$$P\left(x>b\right)=P(x>c).$$

4. A factory decided to install a voltage regulator to compensate for variations in the voltage of the local network. Consider v the variable that characterizes output voltage of the regulator. In particular, the voltage regulator works well if the temperature t degrees (in Celsius) lies in the interval [10, 40]. For this reason, when the temperature is between 10 and 40 degrees the output voltage v can be considered constant and equal to V0>0. If the temperature is not within this interval, the output voltage v can be modeled by a Gaussian random variable with parameters m = V0 and σ = V0/4.

a) Determine and sketch the probability density function of the random variable v.

b) Consider that the temperature t can be modeled by a Gaussian random variable with parameters m = 30 and σ= 5, compute the probability that output voltage is less than V0/2.

5. Consider the joint probability density function given by

$$p\_{xy}\left(X,Y\right)=\left\{\begin{matrix}\frac{1}{π}, X^{2}+Y^{2}\leq 1\\0, X^{2}+Y^{2}>1\end{matrix}\right.$$

a) Compute $p\_{x|y=Y}\left(X\right).$

b) Are x and y are statistically independent random variables?

c) Compute the probability that x is negative when y is positive.

d) Calculate the probability that x is greater than y.

6. An electronics factory produces two types of sensors: A e B. Consider that the lifetime of sensors A and B can be modeled by the random variables x and y, respectively. Now suppose that through experiments we have found that the joint probability density function of x and y is given by

 $p\_{xy}\left(X,Y\right)=\frac{10^{-4}}{6}e^{-\left(\frac{X}{200}+\frac{Y}{300}\right)}u\left(X\right)u(Y)$.

a) Determine the probability that sensor B fails before sensor A.

b) Are the random variables x and y statistically independent? Explain in detail.

7. The joint probability density function of 2 random variables x and y is described by

$$p\_{xy}\left(X,Y\right)=g\left(X\right)h\left(Y\right).$$

a) Determine $p\_{x}\left(X\right)$ and $p\_{y}\left(Y\right)$.

b) Are the random variables x and y statistically independent? Explain in detail.

8. A factory yields two types of devices. Type A devices occur with probability α and work for a relatively short time that is geometrically distributed with parameter r. Type B devices work much longer, occur with probability 1- α and have a lifetime that is geometrically distributed with parameter s. Let X be the lifetime of an arbitrary device.

a) Write down the conditional probabilities of X given the type of device.

b) Find the probability mass function of X.