Adaptive Minimum BER Reduced-Rank Interference Suppression for Very Large Multiuser MIMO Systems

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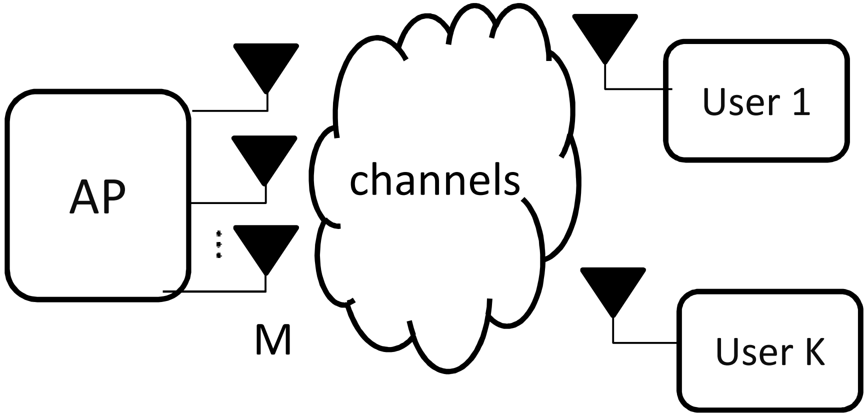
Introduction

- Massive MIMO systems present many design challenges for wireless engineers due to the large number of parameters involved for estimation.
- In this context, a wireless receiver must deal with a number of parameters that ranges from dozens to hundreds .
- Reduced-rank techniques are amongst the most suitable methods to deal with large systems.
  - Eigen decomposition (EIG) [1]
  - Multistage Wiener filter (MWF) [2, 3]
  - Auxiliary vector filtering (AVF) [4]
  - Joint iterative optimisation (JIO) techniques [5]-[7].
- We propose reduced-rank algorithms based on the JIO concept that minimise the BER for massive MIMO systems.

Signal Model



- Uplink MU-MIMO signal model:

$$\mathbf{r}(i) = \sum_{k=1}^{K} A_k \mathbf{h}_k(i) b_k(i) + \mathbf{n}(i),$$

where
h(i) is the M x 1 channel vector ,
n(i) is an M x 1 the noise vector,
(i) denotes the time instant,
A_k is the amplitude of user k,
and M is the number of receive antennas.

- The signal processing scheme observes r(i) and performs linear filtering.

Design of MBER Reduced-Rank Schemes

- A Dx1 projected received vector is obtained as follows

$$\bar{\mathbf{r}}(i) = \mathbf{S}_D^H \mathbf{r}(i),$$

- The filter output is $\bar{x}_k(i) = \bar{\mathbf{w}}_k^H \bar{\mathbf{r}}(i) = \bar{\mathbf{w}}_k^H \mathbf{S}_D^H \mathbf{r}(i).$
- By minimizing the error probability for user k, which is given by [6]

$$P_e = P(\tilde{x}_k < 0) = \int_{-\infty}^{0} f(\tilde{x}_k) d\tilde{x}_k$$
$$= Q\left(\frac{\mathrm{sign}\{b_k(i)\}\Re[\bar{x}_k(i)]}{\rho(\bar{\mathbf{w}}_k^H \mathbf{S}_D^H \mathbf{S}_D \bar{\mathbf{w}}_k)^{\frac{1}{2}}} \right),$$

with respect to the filter and rank-reduction matrix, we obtain the gradients, where $\tilde{x}_k = \mathrm{sign}\{b_k(i)\}\Re[\bar{x}_k(i)]$, ρ is the radius parameter of the kernel density estimate [8].

Design of MBER Reduced-Rank Schemes (cont.)

- The gradient expressions are given by

$$\frac{\partial P_e}{\partial \bar{\mathbf{w}}_k^*} == \frac{-\exp\left(\frac{-|\Re[\bar{x}_k(i)]|^2}{2\rho^2 \bar{\mathbf{w}}_k^H \mathbf{S}_D^H \mathbf{S}_D \bar{\mathbf{w}}_k}\right)\text{sign}\{b_k(i)\}}{2\sqrt{2\pi}\rho} \times \left(\frac{\mathbf{S}_D^H \mathbf{r}}{(\bar{\mathbf{w}}_k^H \mathbf{S}_D^H \mathbf{S}_D \bar{\mathbf{w}}_k)^{\frac{1}{2}}} - \frac{\Re[\bar{x}_k(i)]\mathbf{S}_D^H \mathbf{S}_D \bar{\mathbf{w}}_k}{(\bar{\mathbf{w}}_k^H \mathbf{S}_D^H \mathbf{S}_D \bar{\mathbf{w}}_k)^{\frac{3}{2}}}\right).$$

$$\frac{\partial P_e}{\partial \mathbf{S}_D^*} = \frac{-\exp\left(\frac{-|\Re[\bar{x}_k(i)]|^2}{2\rho^2 \bar{\mathbf{w}}_k^H \mathbf{S}_D^H \mathbf{S}_D \bar{\mathbf{w}}_k}\right)\text{sign}\{b_k(i)\}}{2\sqrt{2\pi}\rho} \times \left(\frac{\mathbf{r}\bar{\mathbf{w}}_k^H}{(\bar{\mathbf{w}}_k^H \mathbf{S}_D^H \mathbf{S}_D \bar{\mathbf{w}}_k)^{\frac{1}{2}}} - \frac{\mathbf{S}_D \bar{\mathbf{w}}_k \bar{\mathbf{w}}_k^H \Re[\bar{x}_k(i)]}{(\bar{\mathbf{w}}_k^H \mathbf{S}_D^H \mathbf{S}_D \bar{\mathbf{w}}_k)^{\frac{3}{2}}}\right).$$

Proposed MBER Adaptive Algorithms

- The algorithm is given by

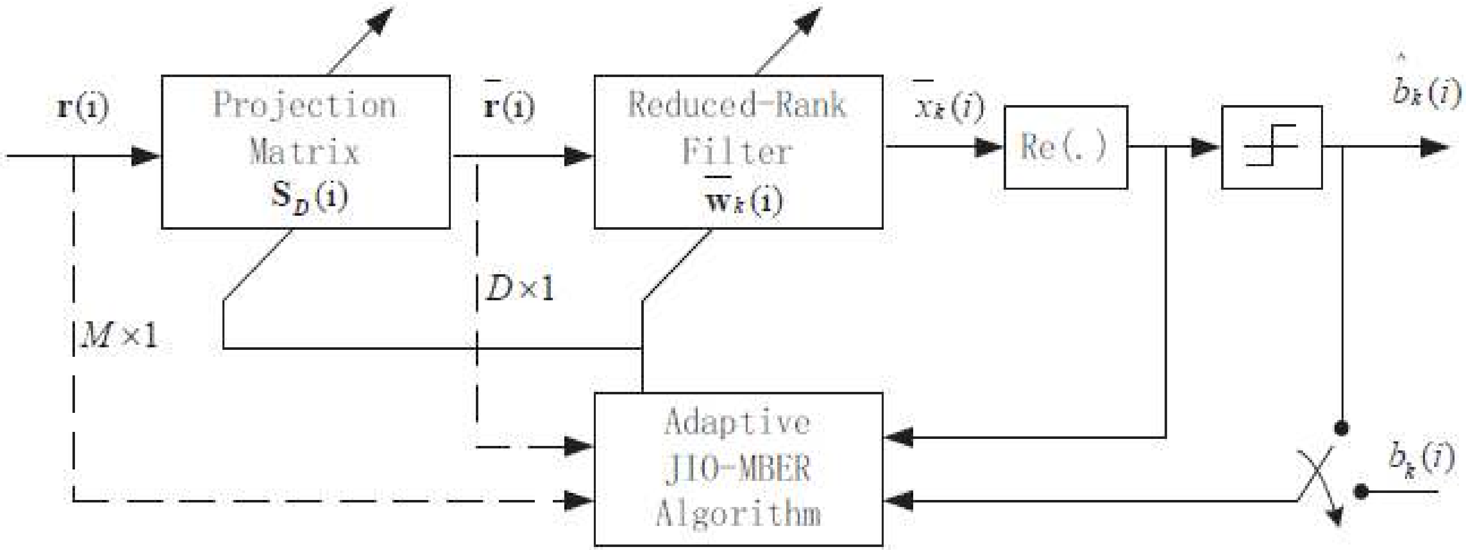
Initialize $\bar{\mathbf{w}}_k(0)$ and $\mathbf{S}_D(0)$.
Set step-size values μ_w and μ_{S_D}
for each time instant $i = 0, 1, \cdots$ do
    Compute $\bar{\mathbf{w}}_k(i+1)$ and $\mathbf{S}_D(i+1)$ using the following update expressions.
    Scale the $\bar{\mathbf{w}}_k$ using $\bar{\mathbf{w}}_k = \frac{\bar{\mathbf{w}}_k}{\sqrt{\bar{\mathbf{w}}_k^H \mathbf{S}_D^H \mathbf{S}_D \bar{\mathbf{w}}_k}}$.
Obtain $\bar{\mathbf{w}}_k(i+1)$ and $\mathbf{S}_D(i+1)$ for the next time instant.

- where the update expressions for the reduced-rank filter and the rank-reduction matrix are given by

$$\bar{\mathbf{w}}_k(i+1) = \bar{\mathbf{w}}_k(i) + \mu_w \frac{\exp\left(\frac{-|\Re[\bar{x}_k(i)]|^2}{2\rho^2}\right)\mathrm{sign}\{b_k(i)\}}{2\sqrt{2\pi}\rho} \times \left(\mathbf{S}_D^H(i)\mathbf{r}(i) - \Re[\bar{x}_k(i)]\mathbf{S}_D^H(i)\mathbf{S}_D(i)\bar{\mathbf{w}}_k(i)\right)$$

Proposed MBER Adaptive Algorithms (cont.)

$$\mathbf{S}_D(i+1) = \mathbf{S}_D(i) + \mu_{S_D} \frac{\exp\left(\frac{-|\Re[\bar{x}_k(i)]|^2}{2\rho^2}\right)\operatorname{sign}\{b_k(i)\}}{2\sqrt{2\pi}\rho} \times \left(\mathbf{r}(i)\bar{\mathbf{w}}_k^H(i) - \mathbf{S}_D(i)\bar{\mathbf{w}}_k(i)\bar{\mathbf{w}}_k^H(i)\Re[\bar{x}_k(i)]\right)$$



Computational Complexity

Algorithm	Number of operations per symbol	
	Multiplications	Additions
Full-Rank-LMS	$2M+1$	$2M$
Full-Rank-MBER	$4M+1$	$4M-1$
MWF-LMS	DM^2-M^2 $+2DM+4D+1$	DM^2-M^2 $+3D-2$
EIG	$O(M^3)$	$O(M^3)$
JIO-LMS	$3DM+M$ $+3D+6$	$2DM+M$ $+4D-2$
MWF-MBER	$(D+1)M^2$ $+(3D+1)M+3D$ $+M+10$	$(D-1)M^2$ $+(2D-1)M$ $+2D+M+1$
JIO-MBER	$6MD+5D$ $+M+11$	$5MD+D$ $-M-1$

Automatic Rank Selection Mechanism

- Since the performance of reduced-rank algorithms depends on the rank, we develop a rank adaptation algorithm based on the error probability,

$$P_D(i) = Q\left(\frac{\text{sign}\{b_k(i)\}\Re[\bar{x}_k^D(i)]}{\rho}\right)$$

- For each time instant, we adapt a reduced-rank filter and a rank-reduction matrix with the maximum allowed rank, which can be expressed as

$$\bar{\tilde{\mathbf{w}}}_k(i) = [\bar{\tilde{w}}_1(i), \ldots, \bar{\tilde{w}}_{D_{\min}}(i), \ldots, \bar{\tilde{w}}_{D_{\max}}(i)]^T$$

$$\tilde{\mathbf{S}}_D(i) = \begin{bmatrix} \tilde{s}_{1,1}(i) & \ldots & \tilde{s}_{1,D_{\min}}(i) & \ldots & \tilde{s}_{1,D_{\max}}(i) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \tilde{s}_{M,1}(i) & \ldots & \tilde{s}_{M,D_{\min}}(i) & \ldots & \tilde{s}_{M,D_{\max}}(i) \end{bmatrix}$$

Automatic Rank Selection Mechanism (cont.)

- For each symbol, we test the value of the rank D within a range.
- For each tested rank, we substitute the filter

$$\tilde{\bar{\mathbf{w}}}_k'(i) = [\tilde{\bar{w}}_1(i), \ldots, \tilde{\bar{w}}_D(i)]^T$$

and the matrix

$$\tilde{\mathbf{S}}_D'(i) = \begin{bmatrix} \tilde{s}_{1,1}(i) & \ldots & \tilde{s}_{1,D}(i) \\ \vdots & \vdots & \vdots \\ \tilde{s}_{M,1}(i) & \ldots & \tilde{s}_{M,D}(i) \end{bmatrix}$$

to obtain the probability of error.

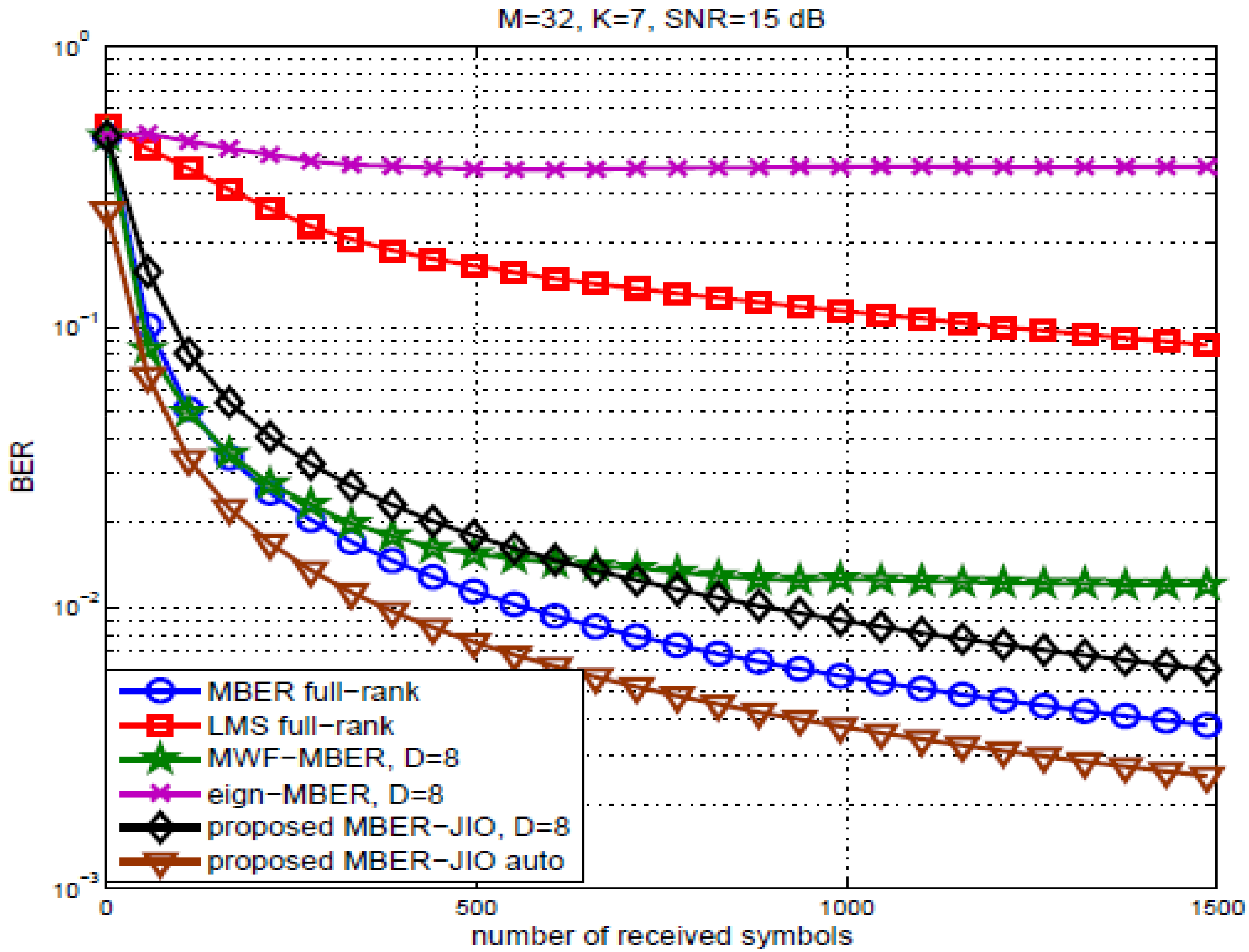
The optimum rank can be selected as

$$D_{\text{opt}}(i) = \arg \min_{D \in \{D_{min}, \ldots, D_{max}\}} P_D(i).$$

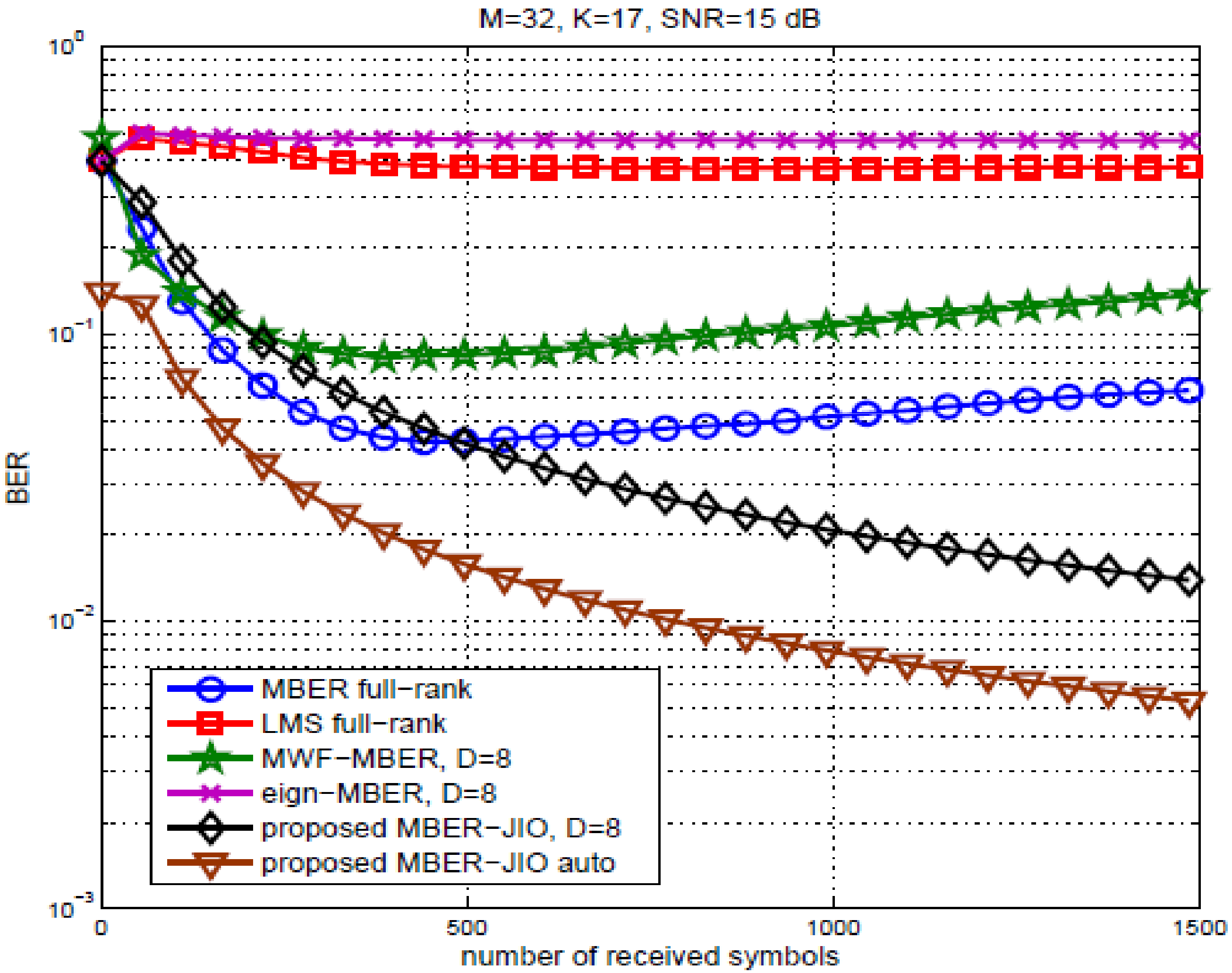
Simulations

- The number of receive antennas at the BS is M=32. The channel coefficient is computed according to Clarke's model [8]
- We optimized the parameters of the JIO-MBER adaptive reduced-rank SG algorithms with step sizes 0.01 and 0.025.
- The step sizes for LMS adaptive full rank, SG adaptive MBER full rank and the conventional adaptive reduced-rank techniques are 0*.085, 0.05 and 0.035,* respectively.
- The initial full rank and reduced-rank filters are all zero vectors. The algorithms process 250 symbols in TR and 1500 symbols in DD.

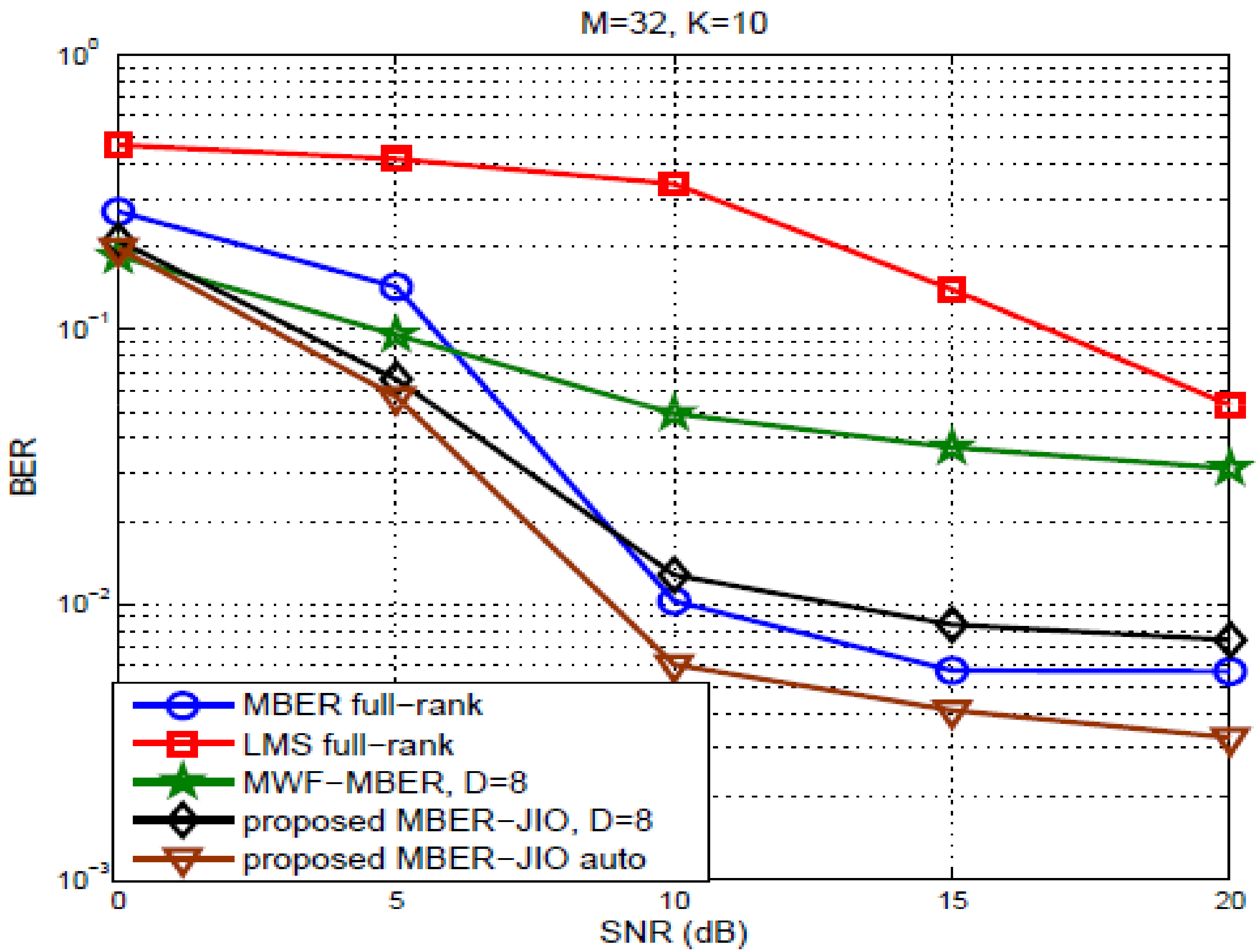
(1) BER X Symbols, (Dmin=3, Dmax=20, K=7) normalized Doppler freq. 0.00001



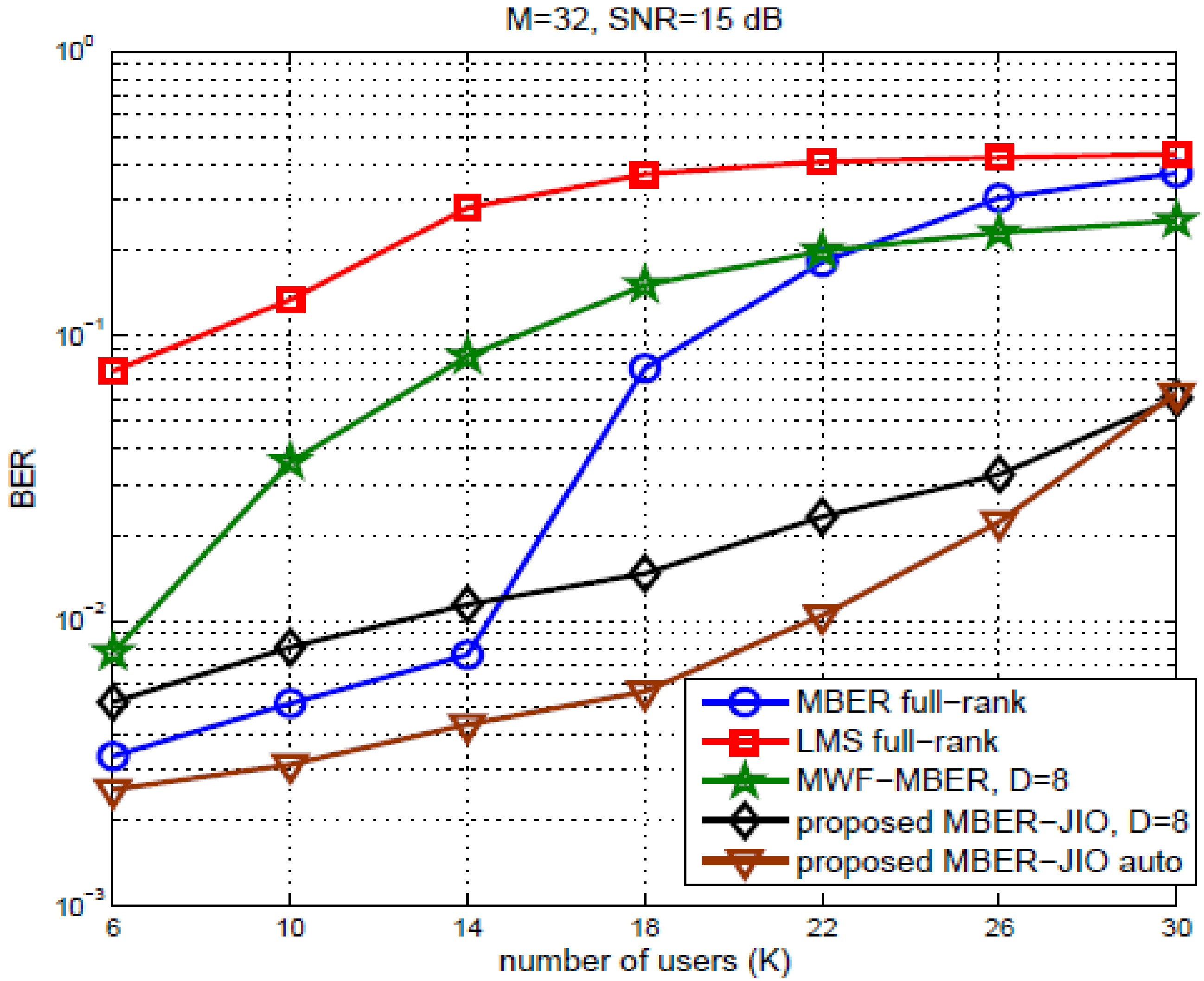
(2) BER X Symbols, (Dmin=3, Dmax=20, K=17), normalized Doppler freq. 0.00001



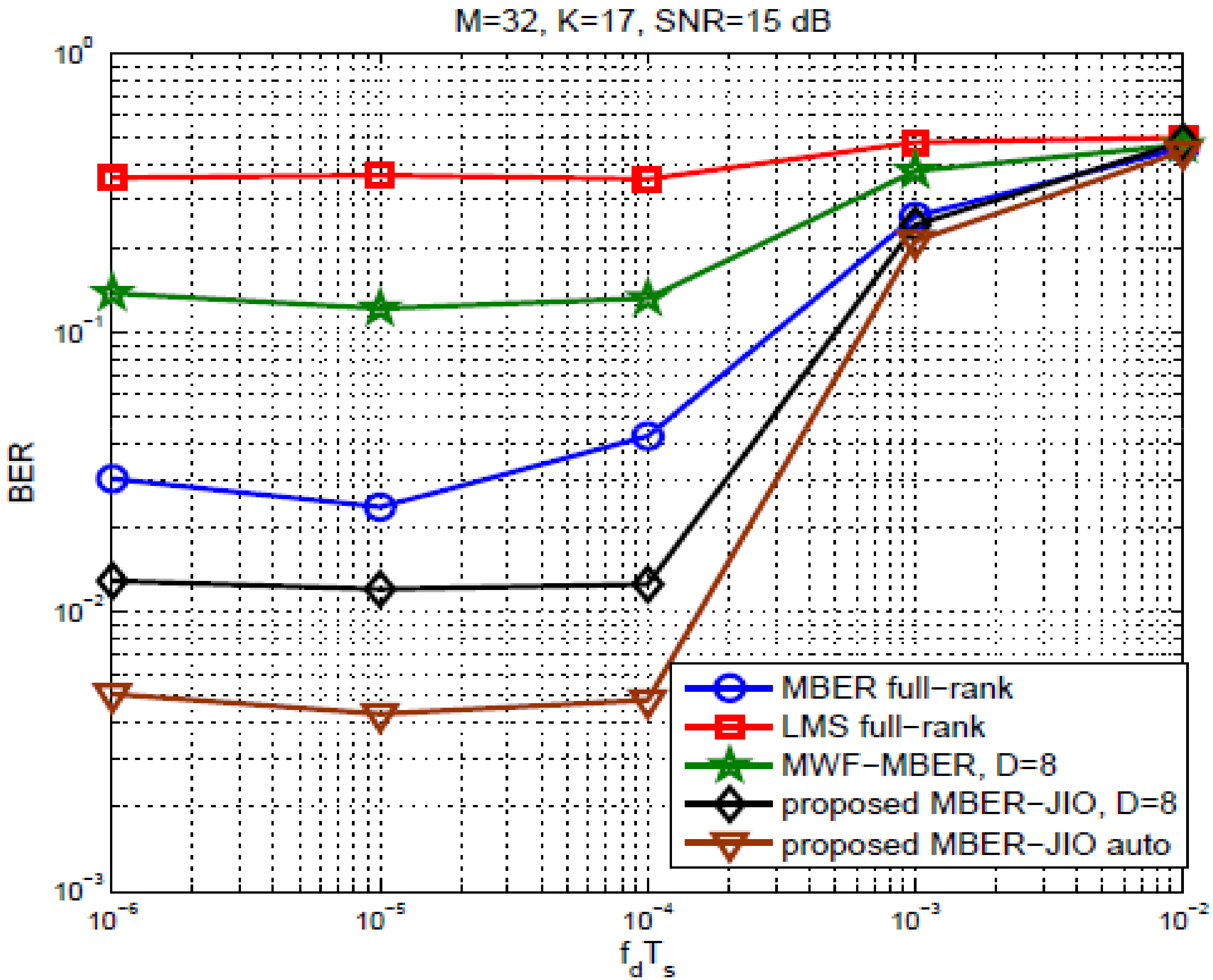
(3) BER X SNR, (Dmin=3, Dmax=20, K=10), normalized Doppler freq. 0.00001



(4) BER X number of users,(Dmin=3, Dmax=20, SNR=15 dB), normalized Doppler freq. 0.00001



(5) BER X channel fading rate (Dmin=3, Dmax=20, SNR=15 dB, K=17)



Conclusions

- A novel adaptive MBER reduced-rank scheme based on joint iterative optimization of filters has been proposed for multiuser MIMO systems with a large number of antennas.

- We have developed SG based algorithms for the adaptive estimation of the reduced-rank receive filter and the rank-reduction matrix.

- An automatic rank selection scheme using the BER as a criterion has also been devised.

- The simulation results have shown that the proposed JIO-MBER adaptive reduced-rank algorithms significantly outperform the existing full-rank and reduced-rank algorithms at a low cost.

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