P1 2015.1

Q1 A e B são 2 eventos tais que $P(A) = 1/4$, $P(B|A) = 1/2$, $P(A|B) = 1/4$

a) $P(A\cap B) = P(B|A).P(A) = 1/8 \neq 0$

Falso porque $P(A\cap B) \neq 0 \rightarrow$ os eventos A e B não são mutuamente exclusivos.

b) $A\subset B = A\cap B = A \rightarrow P(A\cap B) = P(A)$

Falso, porque $\underbrace{P(A\cap B)}_{1/8} \neq \underbrace{P(A)}_{1/4}$

c) $P(A\cap B) = P(A).P(B) = 1/4 \cdot 1/2 = 1/8$

Verdadeiro

d) $P(B) = 3/4 \rightarrow P(A\cap B) = P(A|B).P(B) \rightarrow P(B) = \dfrac{P(A\cap B)}{P(A|B)} = \dfrac{1}{8}\cdot\dfrac{4}{1} = 1/2$

Falso

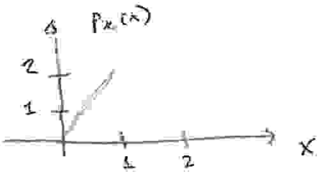
e) $P(\bar{A}|\bar{B}) = \dfrac{P(\bar{A}\cap\bar{B})}{P(\bar{B})} = \dfrac{P(\bar{A}\cap\bar{B})}{1-P(B)} = \dfrac{P(\bar{A}\cap\bar{B})}{1/2} = \dfrac{1-(\overbrace{P(A)}^{1/4}+\overbrace{P(B)}^{1/2}-\overbrace{P(A\cap B)}^{1/8})}{1/2} = \dfrac{1-5/8}{1/2} = \dfrac{3}{8}\cdot\dfrac{2}{1} = \dfrac{3}{4}$

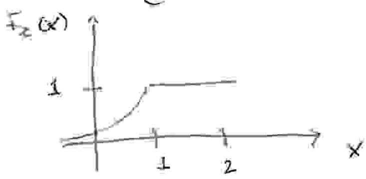
Verdadeiro

Q2 $p_X(x) = \begin{cases} kx, & 0<x<1 \\ 0, & \text{caso contrário} \end{cases}$

a) $\int_0^1 k\,x\,dx = 1 = k\left[\dfrac{x^2}{2}\right]_0^1 = \dfrac{k}{2} \rightarrow \boxed{k=2} \rightarrow p_X(x) = \begin{cases} 2x, & 0<x<1 \\ 0, & \text{caso contrário} \end{cases}$

b) $F_X(x) = \int_{-\infty}^{\infty} p_X(x)\,dx = \begin{cases} 0, & x<0 \\ \int_0^x 2u\,du = x^2, & 0\le x<1 \\ \int_0^1 2u\,du = 1, & 1\le x \end{cases} = \begin{cases} 0, & x<0 \\ x^2, & 0\le x<1 \\ 1, & 1\le x \end{cases}$





c) $P(1/4 < x < 2) = F_X(2) - F_X(1/4) = 1 - (1/4)^2 = \dfrac{15}{16}$

d) $g_X(x) = \dfrac{1}{\sqrt{2\pi}} e^{-(x^2-x+a)}$

$\int_{-\infty}^{\infty} g_X(x) = \int_{-\infty}^{\infty} \dfrac{1}{\sqrt{2\pi}} e^{-(x^2-x+a)} = \int_{-\infty}^{\infty} \dfrac{1}{\sqrt{2\pi}} e^{-(x^2-x+1/4+a-1/4)}\,dx = 1$

$= \int_{-\infty}^{\infty} e^{-(a-1/4)} \dfrac{1}{\sqrt{2\pi}} e^{-(x^2-x+1/4)}\,dx = 1$

$= e^{-(a-1/4)} \underbrace{\int_{-\infty}^{\infty} \dfrac{1}{\sqrt{2\pi}} e^{-(x-1/2)^2}\,dx}_{1} = 1$

$e^{-(a-1/4)} = 1 \rightarrow a - 1/4 = 0 \rightarrow \boxed{a = 1/4}$

Q3 $\quad p_{xy}(x,y) = \begin{cases} c\,xy, & 0 < x < 1,\ 0 < y < 1 \\ 0 \end{cases}$

a) $$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_{xy}(x,y)\,dx\,dy = 1$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} c\,xy\,dx\,dy = \int_0^1\int_0^1 c\,xy\,dx\,dy = \int_0^1 c\left[\frac{x^2}{2}\right]_0^1 y\,dy$$

$$= \frac{c}{2}\int_0^1 y\,dy = \frac{c}{4}\left[y^2\right]_0^1 = \frac{c}{4} = 1 \rightarrow \boxed{c=4}$$

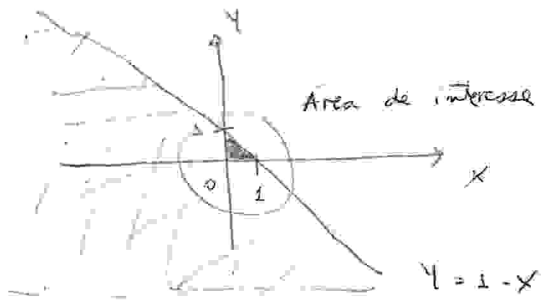
b) $$p_x(x) = \int_{-\infty}^{\infty} p_{xy}(x,y)\,dy = \int_0^1 4xy\,dy = \left[2xy^2\right]_0^1 = \begin{cases} 2x, & 0 < x < 1 \\ 0, & c.c \end{cases}$$

$$p_y(y) = \int_{-\infty}^{\infty} p_{xy}(x,y)\,dx = \int_0^1 4xy\,dx = \left[2x^2y\right]_0^1 = \begin{cases} 2y, & 0 < y < 1 \\ 0, & c.c \end{cases}$$

$p_{xy}(x,y) = p_x(x)\,p_y(y)$ $\rightarrow$ Logo, as v.a.s são est. ind.

$y < 1-x$

c) $$P(x+y<1) = \int_0^1\int_0^{1-y} 4xy\,dx\,dy$$



$$= \int_0^1 4y\left[\frac{x^2}{2}\right]_0^{1-y} dy$$

$$= \int_0^1 2y(1-y)^2\,dy$$

$0 < x < 1$

$0 < y < 1$

$$= \int_0^1 2y(1-2y+y^2)\,dy = \int_0^1 2y - 4y^2 + 2y^3\,dy$$

$$= \left[\frac{2y^2}{2} - \frac{4y^3}{3} + \frac{2}{4}y^4\right]\Big|_0^1 =$$

$$= 1 - \frac{4}{3} + \frac{1}{2} = -\frac{1}{3} + \frac{1}{2} = \frac{1}{6}$$