Adaptive and Array Signal Processing/Processamento de Sinais Adaptativo

CETUC/PUC-Rio - Prof. Rodrigo de Lamare

Tutorial Questions/Lista de Exercícios - 4

1. Suppose that the input to an adaptive linear predictor is white noise with an autocorrelation sequence $R\left(i\right)=σ\_{x}^{2}δ\left(i\right).$

a) Solve the normal equations and find the optimum k-th order one-step linear predictor $w$.

b) Minimize the mean-square prediction error using the method of steepest descent with a step size $μ=1/(5σ\_{x}^{2})$ and an initial weight vector $w\_{0}=[1,1,\cdots ,1]^{T}$ . Does the method of steepest descent converge to the solution found in part (a)?

2. Newton´s method is an iterative algorithm that may be used to find the minimum of a nonlinear function. Applied to the minimization of the mean-square error

$$MSE\left[i\right]=E[e^{2}[i]]$$

where $e\left[i\right]= d\left[i\right]-w^{T}x\left[i\right],$ Newton´s method is

$$w\left[i+1\right]=w\left[i\right]-\frac{1}{2}R^{-1}∇MSE[i]$$

where is the correlation matrix of the the observed data . Introducing a step size $μ$, Newton´s method becomes

$$w\left[i+1\right]=w\left[i\right]-\frac{1}{2}μR^{-1}∇MSE[i]$$

Comparing this to the steepest descent algorithm, we see that the step size $μ$ is replaced with a matrix, $μR^{-1}$ , which alters the descent direction.

a) For what values of $μ$ is Newton´s method stable, i.e., for what values of $μ$ will $w\left[i\right] $converge?

b) What is the optimum value of $μ$, i.e., for what value of $μ$ is the convergence the fastest?

c) Suppose that we form a version of Newton´s method by replacing the gradient with an instantaneous estimate

$$\hat{∇}MSE\left[i\right]=∇e^{2}[i]$$

Derive the coefficient update equation that results from using this gradient estimate and describe how it differs from the LMS algorithm.

d) Derive an expression that describes the time evolution of $E[w\left[i\right]]$ using the LMS Newton algorithm derived in part c).