Adaptive reduced-rank LCMV beamforming algorithms based on joint iterative optimisation of filters

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A novel approach to linearly constrained minimum variance (LCMV) beamforming based on reduced-rank processing is proposed. The method is based on a constrained joint iterative optimisation of an adaptive projection matrix and a reduced-rank filter according to the minimum variance criterion. We derive LCMV expressions for the design of the projection matrix and the reduced-rank filter and present low-complexity adaptive algorithms for their efficient implementation. Simulations show that the proposed scheme outperforms the full-rank and existing reduced-rank methods with low complexity.

Introduction: Adaptive beamforming techniques have attracted considerable interest and found applications in radar, wireless communications and sonar, amongst others. The optimal linearly constrained minimum variance (LCMV) beamformer minimises the array output power while maintaining a constant response in the direction of the signal of interest (SOI) [1]. However, this technique requires the inversion of the input data covariance matrix $\boldsymbol{R}$ and knowledge of the steering vector. Adaptive versions of the LCMV beamformer were reported with stochastic gradient (LCMV-SG) [2] and recursive least squares (LCMV-RLS) [3, 4] algorithms. The convergence and tracking performances of these algorithms are affected by the number of interferers, the eigenvalue spread of $\boldsymbol{R}$ and the number of sensor elements M [5].

A key technique in short data-record situations and in problems with many parameters is reduced-rank adaptive filtering. Prior work considered several methods: eigen-decomposition [5], the multi-stage Wiener filter (MSWF) [6, 7] and the auxiliary-vector filter (AVF) algorithms [8]. Despite improved convergence and tracking performance, these methods are complex and suffer from numerical problems.

In this Letter we propose a novel adaptive reduced-rank LCMV beamforming approach based on constrained joint iterative optimisation of adaptive filters.

System model: Let us consider a smart antenna system equipped with a uniform linear array (ULA) of M omnidirectional sensors. The signals of K narrowband sources impinge on the array ($K < M$) with unknown directions of arrival (DOA). The ith snapshot's $M \times 1$ vector of sensor array outputs can be modelled as [1]

$$\boldsymbol{r}(i) = \boldsymbol{A}(\boldsymbol{\theta})\boldsymbol{s}(i) + \boldsymbol{n}(i), \quad i = 1, \ldots, N \tag{1}$$

where $\boldsymbol{\theta} = [\theta_0, \ldots, \theta_{K-1}]^T \in \mathcal{C}^{K\times 1}$ is the vector of the unknown DOA, $\boldsymbol{A}(\boldsymbol{\theta}) = [\boldsymbol{a}(\theta_0), \ldots, \boldsymbol{a}(\theta_{K-1})] \in \mathcal{C}^{M\times K}$ is the complex matrix with the steering vectors $\boldsymbol{a}(\theta_k) = [1,\ e^{-2\pi j(d/\lambda_c)\cos\theta_k}, \ldots, e^{-2\pi j(M-1)(d/\lambda_c)\cos\theta_k}]^T \in \mathcal{C}^{M\times 1}$, $(k = 0, \ldots, K-1)$, where $(\cdot)^T$ denotes transpose, λ_c is the wavelength, $d = \lambda_c/2$ is the inter-element spacing of the ULA, $\boldsymbol{s}(i) \in \mathcal{C}^{K\times 1}$ is the complex vector of the source signals, $\boldsymbol{n}(i) \in \mathcal{C}^{M\times 1}$ is the vector of zero-mean Gaussian noise with variance σ^2 and N is the number of snapshots.

Problem statement: The LCMV filter $\boldsymbol{w}(i) = [w_1^{(i)}\ w_2^{(i)} \ldots w_M^{(i)}]^T$, solves the following optimisation problem

$$\begin{aligned} &\text{minimise}\ \ E[|\boldsymbol{w}^H(i)\boldsymbol{r}(i)|^2] = \boldsymbol{w}^H(i)\boldsymbol{R}\boldsymbol{w}(i) \\ &\text{subject to}\ \ \boldsymbol{w}^H(i)\boldsymbol{a}(\theta_k) = 1 \end{aligned} \tag{2}$$

where $\boldsymbol{R} = E[\boldsymbol{r}(i)\boldsymbol{r}^H(i)]$, $(\cdot)^H$ denotes Hermitian transpose and $E[\cdot]$ is the expected value. The filter $\boldsymbol{w}(i)$ can be estimated via SG or RLS algorithms [5], however their convergence speed depends on M and the eigenvalue spread of $\boldsymbol{R}$. A reduced-rank algorithm attempts to circumvent these limitations by reducing the number of adaptive coefficients and extracting the most important features of the processed data. Let us consider an $M \times D$ projection matrix $\boldsymbol{S}_D(i)$ which carries out a dimensionality reduction on the received data as given by

$$\bar{\boldsymbol{r}}(i) = \boldsymbol{S}_D^H(i)\boldsymbol{r}(i) \tag{3}$$

where, in what follows, all D-dimensional quantities are denoted with a 'bar'. The reduced-rank vector $\bar{\boldsymbol{r}}(i)$ is the input to a filter represented by the $D \times 1$ vector $\bar{\boldsymbol{w}}(i) = [\bar{w}_1^{(i)}\ \bar{w}_2^{(i)} \ldots \bar{w}_D^{(i)}]^T$. The filter output is

$$x(i) = \bar{\boldsymbol{w}}^H(i)\bar{\boldsymbol{r}}(i) \tag{4}$$

If we consider the LCMV design in (2) with the reduced-rank covariance matrix $\bar{\boldsymbol{R}} = E[\bar{\boldsymbol{r}}(i)\bar{\boldsymbol{r}}^H(i)] = \boldsymbol{S}_D^H(i)\boldsymbol{R}\boldsymbol{S}_D(i)$ and the reduced-rank steering vector $\bar{\boldsymbol{a}}(\theta_k) = \boldsymbol{S}_D^H(i)\boldsymbol{a}(\theta_k)$ we obtain

$$\bar{\boldsymbol{w}}(i) = (\bar{\boldsymbol{a}}^H(\theta_k)\bar{\boldsymbol{R}}^{-1}\bar{\boldsymbol{a}}(\theta_k))^{-1}\bar{\boldsymbol{R}}^{-1}\bar{\boldsymbol{a}}(\theta_k) \tag{5}$$

The minimum variance (MV) for a LCMV filter with rank D is

$$\text{MV} = (\mathbf{a}^H(\theta_k)\mathbf{S}_D(i)(\mathbf{S}_D^H(i)\mathbf{R}\mathbf{S}_D(i))^{-1}\mathbf{S}_D^H(i)\mathbf{a}(\theta_k))^{-1} \tag{6}$$

Based on the development above, the problem is how to efficiently and/or optimally design an $M \times D$ transformation matrix $\boldsymbol{S}_D(i)$.

Proposed method: In the proposed method, the projection matrix $\boldsymbol{S}_D(i)$ and the reduced-rank filter $\bar{\boldsymbol{w}}(i)$ are jointly optimised according to the LCMV criterion to yield a scalar output $x(i)$. The projection matrix $\boldsymbol{S}_D(i)$ is structured as a bank of D full-rank filters (filters with the same dimensions of $\boldsymbol{r}(i)$, i.e. $M \times 1$) $s_j(i) = [s_{1,j}(i)\ s_{2,j}(i) \ldots s_{M,j}(i)]^T$ $(j = 1, \ldots, D)$ with dimensions $M \times 1$ as given by $\boldsymbol{S}_D(i) = [\boldsymbol{s}_1(i)|\boldsymbol{s}_2(i)|\ldots|\boldsymbol{s}_D(i)]$. By expressing the output estimate $x(i)$ of the reduced-rank scheme as a function of $\boldsymbol{r}(i)$, $\boldsymbol{S}_D(i)$ and $\bar{\boldsymbol{w}}(i)$ we obtain

$$x(i) = \bar{\boldsymbol{w}}^H(i)\boldsymbol{S}_D^H(i)\boldsymbol{r}(i) = \bar{\boldsymbol{w}}^H(i)\bar{\boldsymbol{r}}(i) \tag{7}$$

Note that for $D = 1$ the proposed scheme becomes a conventional full-rank LCMV filtering scheme with an additional weight parameter $w_D(i)$ that provides a gain on the output. For $D > 1$, the signal processing tasks are changed, $\boldsymbol{S}_D^H(i)$ computes a subspace projection and the reduced-rank filter $\bar{\boldsymbol{w}}^H(i)$ estimates the desired output.

The LCMV expressions for the filters $\boldsymbol{S}_D(i)$ and $\bar{\boldsymbol{w}}(i)$ can be computed via the proposed optimisation problem

$$\begin{aligned} &\text{minimise}\ E[|\bar{\boldsymbol{w}}^H(i)\boldsymbol{S}_D^H(i)\boldsymbol{r}(i)|^2] \\ &\text{subject to}\ \bar{\boldsymbol{w}}^H(i)\boldsymbol{S}_D^H(i)\boldsymbol{a}(\theta_k) = 1 \end{aligned} \tag{8}$$

The constrained optimisation problem in (8) can be transformed by the method of Lagrange multipliers into an unconstrained optimisation problem [5] the cost function of which is

$$\mathcal{L}_{MV} = E[|\bar{\boldsymbol{w}}^H(i)\boldsymbol{S}_D^H(i)\boldsymbol{r}(i)|^2] + 2\Re[\lambda^*(\bar{\boldsymbol{w}}^H(i)\boldsymbol{S}_D^H(i)\boldsymbol{a}(\theta_k) - 1)] \tag{9}$$

where λ is a scalar Lagrange multiplier, * denotes complex conjugate and the operator $\Re[\cdot]$ selects the real part of the argument. By fixing $\bar{\boldsymbol{w}}(i)$, minimising (9) with respect to $\boldsymbol{S}_D(i)$ and solving for λ, we get

$$\begin{aligned} \boldsymbol{S}_D(i) = {} & [\bar{\boldsymbol{w}}^H(i)\boldsymbol{R}_w^{-1}\bar{\boldsymbol{w}}(i)\boldsymbol{a}^H(\theta_k)\boldsymbol{R}^{-1}\boldsymbol{a}(\theta_k)]^{-1} \\ & \boldsymbol{R}^{-1}(i)\boldsymbol{a}(\theta_k)\bar{\boldsymbol{w}}^H(i)\boldsymbol{R}_w^{-1}(i) \end{aligned} \tag{10}$$

where $\boldsymbol{R}(i) = E[\boldsymbol{r}(i)\boldsymbol{r}^H(i)]$ and $\boldsymbol{R}_w(i) = E[\bar{\boldsymbol{w}}(i)\bar{\boldsymbol{w}}^H(i)]$. By fixing $\boldsymbol{S}_D(i)$, minimising (9) with respect to $\bar{\boldsymbol{w}}(i)$ and solving for $\lambda 1$, we arrive at

$$\bar{\boldsymbol{w}}(i) = [\bar{\boldsymbol{a}}^H(\theta_k)\bar{\boldsymbol{R}}^{-1}(i)\bar{\boldsymbol{a}}(\theta_k)]^{-1}\bar{\boldsymbol{R}}^{-1}(i)\bar{\boldsymbol{a}}(\theta_k) \tag{11}$$

where $\bar{\boldsymbol{R}}(i) = E[\boldsymbol{S}_D^H(i)\boldsymbol{r}(i)\boldsymbol{r}^H(i)\boldsymbol{S}_D(i)] = E[\bar{\boldsymbol{r}}(i)\bar{\boldsymbol{r}}^H(i)]$, $\bar{\boldsymbol{a}}(\theta_k) = \boldsymbol{S}_D^H(i)\boldsymbol{a}(\theta_k)$

The associated MV is

$$\text{MV} = (\bar{\boldsymbol{a}}^H(\theta)\bar{\boldsymbol{R}}^{-1}(i)\bar{\boldsymbol{a}}(\theta))^{-1} \tag{12}$$

Note that the filter expressions in (10) and (11) are not closed-form solutions for $\bar{\boldsymbol{w}}(i)$ and $\boldsymbol{S}_D(i)$ since (10) is a function of $\bar{\boldsymbol{w}}(i)$ and (11) depends on $\boldsymbol{S}_D(i)$ and thus it is necessary to iterate (10) and (11) with an initial value to obtain a solution.

Low-complexity implementation: We present a low-complexity adaptive algorithm for efficient implementation of the proposed method and show its computational complexity. By computing the instantaneous gradient terms of (9) with respect to $\boldsymbol{S}_D{}^*(i)$ and $\bar{\boldsymbol{w}}^*(i)$, we get

$$\begin{aligned} \nabla\mathcal{L}_{MV\boldsymbol{S}_D{}^*(i)} &= x^*(i)\boldsymbol{r}(i)\bar{\boldsymbol{w}}^H(i) + 2\lambda^*\boldsymbol{a}(\theta_k)\bar{\boldsymbol{w}}^H(i) \\ \nabla\mathcal{L}_{MV\bar{\boldsymbol{w}}^*(i)} &= x^*(i)\boldsymbol{S}_D^H(i)\boldsymbol{r}(i) + 2\lambda^*\boldsymbol{S}_D^H(i)\boldsymbol{a}(\theta_k) \end{aligned} \tag{13}$$

By introducing the positive step sizes μ_s and μ_w, using the gradient rules $\boldsymbol{S}_D(i+1) = \boldsymbol{S}_D(i) - \mu_s \nabla \mathcal{L}_{MV\bar{\boldsymbol{S}}_D{}^*(i)}$ and $\bar{\boldsymbol{w}}(i+1) = \bar{\boldsymbol{w}}(i) - \mu_w \nabla \mathcal{L}MV\bar{\boldsymbol{w}}^*(i)$, enforcing the constraint and solving the resulting equations, we obtain

$$\boldsymbol{S}_D(i+1) = \boldsymbol{S}_D(i) - \mu_s x^*(i)[\boldsymbol{r}(i)\bar{\boldsymbol{w}}^H(i) - \boldsymbol{a}(\theta_k)\bar{\boldsymbol{w}}^H(i)\boldsymbol{a}^H(\theta_k)\boldsymbol{r}(i)] \quad (14)$$

$$\bar{\boldsymbol{w}}(i+1) = \bar{\boldsymbol{w}}(i) - \mu_w x^*(i)[\boldsymbol{I} - (\bar{\boldsymbol{a}}^H(\theta_k)\bar{\boldsymbol{a}}(\theta_k))^{-1}\bar{\boldsymbol{a}}(\theta_k)\bar{\boldsymbol{a}}^H(\theta_k)]\bar{\boldsymbol{r}}(i) \quad (15)$$

where $x(i) = \bar{\boldsymbol{w}}^H(i)\boldsymbol{S}_D^H(i)\boldsymbol{r}(i)$. The proposed scheme trades-off a full-rank filter against D full-rank adaptive filters as the projection matrix $\boldsymbol{S}_D(i)$ and one reduced-rank adaptive filter $\bar{\boldsymbol{w}}(i)$ operating simultaneously and exchanging information. The details on the complexity of the analysed methods are given in Table 1 in terms of multiplications. The proposed algorithms have a complexity D times higher than the LCMV full-rank SG [2] and significantly lower than the other analysed techniques.

Table 1: Computational complexity of LCMV algorithms

Algorithm	Multiplications
Full-rank-SG [2]	$3M+2$
Full-rank-RLS [3]	$6M^2+2M+2$
Proposed-SG	$3DM+M+3D+6$
MSWF-SG [7]	$D(2M^2+5M+7)$
MSWF-RLS [7]	$D(4M^2+2M+3)$
AVF [8]	$D(4M^2+4M+1)+4M+2$

Simulations: A smart antenna system with a ULA containing $M = 32$ sensor elements is considered for assessing the beamforming algorithms. The performance of the proposed scheme and algorithms is compared with existing techniques, namely the full-rank LCMV-SG [2] and LCMV-RLS [3], and the reduced-rank algorithms with $\boldsymbol{S}_D(i)$ designed according to an eigen-decomposition, the MSWF [7], the AVF [8] and the optimal linear beamformer that assumes knowledge of the covariance matrix [1] in terms of signal-to-interference-plus-noise ratio (SINR). For each scenario, 200 runs are used to obtain the curves. In all simulations, the DOA of the SOI is $\theta_d = 20°$, the desired signal power is $\sigma_d^2 = 1$, the signal-to-noise ratio (SNR) is $SNR = \sigma_d^2/\sigma^2$ and we have seven interferers at $-45°$, $-30°$, $-10°$, $0°$, $40°$, $60°$, $75°$ with powers following a log-normal distribution with associated standard deviation 3 dB around the SOI's power level. The parameters of the algorithms are optimised and the filters initialised as $\bar{\mathbf{w}}(0) = [1 \quad 0 \ldots 0]$ and $\boldsymbol{S}_D(0) = [\boldsymbol{I}_D^T \quad \mathbf{0}_{D\times(M-D)}^T]$, where $\mathbf{0}_{D\times M-D}^T$ is a $D \times (M-D)$ matrix with zeros.

We first evaluate the SINR performance of the analysed algorithms against the rank D using optimised parameters (μ_0, ν_0 and forgetting factors λ) for all schemes and $N = 250$ snapshots. The results in Fig. 1*a* indicate that the best rank for the proposed scheme is $D = 4$ (which will be used in the second scenario) and it is very close to the optimal full-rank LCMV filter. Our studies with systems with different sizes show that D is relatively invariant to the system size, which brings considerable computational savings. In practice, the rank D can be adapted to further improve performance.

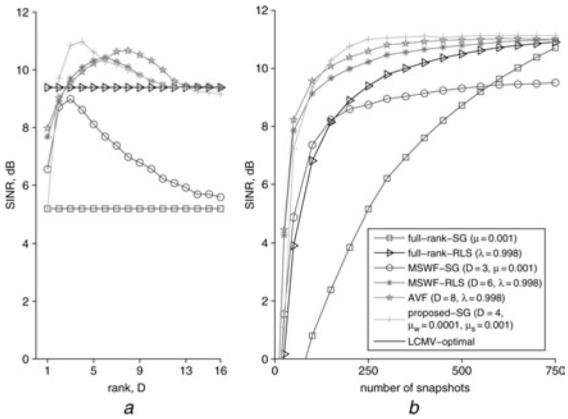

Fig. 1 *SINR performance of LCMV algorithms against rank (D) with $M = 32$, $SNR = 15$ dB, $N = 250$ snapshots (Fig. 1a) and against snapshots with $M = 32$, $SNR = 15$ dB (Fig. 1b)*

We show another scenario in Fig. 1*b* where the adaptive LCMV filters are set to converge to the same level of SINR. The curves show excellent performance for the proposed scheme, which converges much faster than the full-rank-SG algorithm, and is also better than the more complex MSWF-RLS and AVF schemes.

Conclusions: We propose a novel LCMV reduced-rank beamforming scheme based on joint iterative optimisation of adaptive filters. A low-complexity implementation of the proposed scheme is also developed using SG algorithms. The results show performance significantly better than existing schemes with reduced complexity.

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